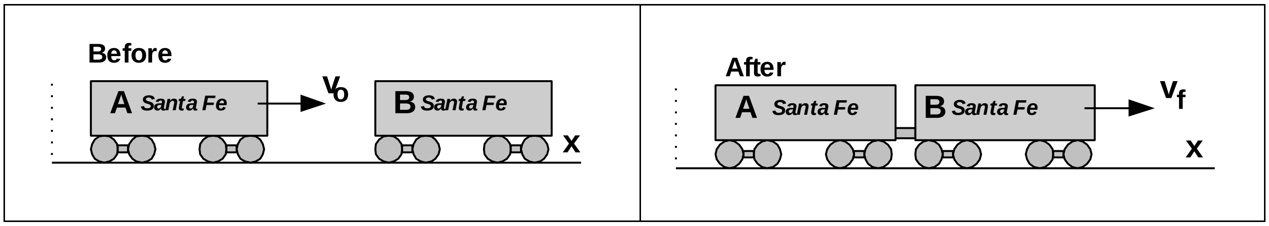
SAMPLE PROBLEMS: 111-SET #8 IMPULSE-MOMENTUM PROBLEMS: 08-1

1) A freight car weighing 25 tons runs into another freight car of the same weight. The first was moving at 6 mi/hr (8.8 ft/sec) and the second was at rest. If the cars are coupled together after the collision, what is their final speed?

Solution: We have a collision problem in 1-dimension. We draw both 'before' and 'after' pictures and select a coordinate system as shown. We have conservation of linear momentum: $\mathbf{p}_{Ai} \oplus \mathbf{p}_{Bi} = \mathbf{p}_{Af} \oplus \mathbf{p}_{Bf}$.



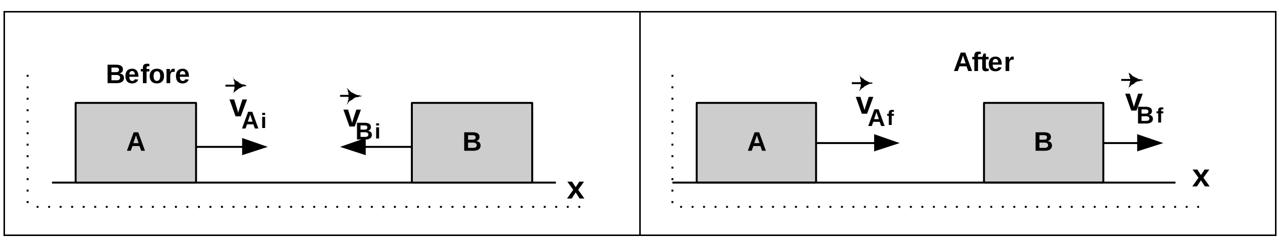
We have a perfectly inelastic collision problem. Hence: $m_1 \mathbf{v}_{1i} \oplus m_2 \mathbf{v}_{2i} = (m_1 + m_2)\, \mathbf{v}_f$

For x-components this becomes: $m_1 v_1 + 0 = (m_1 + m_2)\, v_f$.

Since $m_1 = m_2$, we have: $v_f = (1/2)\, v_1 = 3$ mi/hr (4.4 ft/sec).

2) Two blocks are travelling toward each other. The first has a speed of 10 cm/sec and the second a speed of 60 cm/sec. After the collision the second is observed to be travelling with a speed of 20 cm/sec in a direction opposite to its initial velocity. If the weight of the first block is twice that of the second, determine: (a) the velocity of the first block after collision; (b) whether the collision was elastic or inelastic.

Solution: We have a collision problem in 1-dimension. We draw both 'before' and 'after' pictures and select a coordinate system as shown.



Since the surface is frictionless, and since no work is performed by either mg or the normal, then the net force acting on the system is 0, and we have conservation of linear momentum:

$$\mathbf{p}_{1i} \oplus \mathbf{p}_{2i} = \mathbf{p}_{1i} \oplus \mathbf{p}_{1f}$$

Thus adding the x-components we have: $m_1 v_{1i} - m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

Since $m_1 = 2\, m_2$ we find: $2 v_{1i} - v_{2i} = 2 v_{1f} + v_{2f} \rightarrow (2)(10) - (60) = 2 v_{1f} + 20$

Thus $2\ v_{1f}\ =\ -60$ and $v_{1f}\ =\ -30$ cm/sec ('-' means to left).

The initial KE is given by: $KE_I\ =\ (1/2)\ m_1\ (v_{1I})^2\ +\ (1/2)\ m_2\ (v_{2I})^2$. This gives:

$$=\ (1/2)(2\ m_2)(10)^2\ +(1/2)\ m_2\ (60)^2\ =\ (1/2)(200+3600)\ m_2\ =\ 1900\ m_2$$

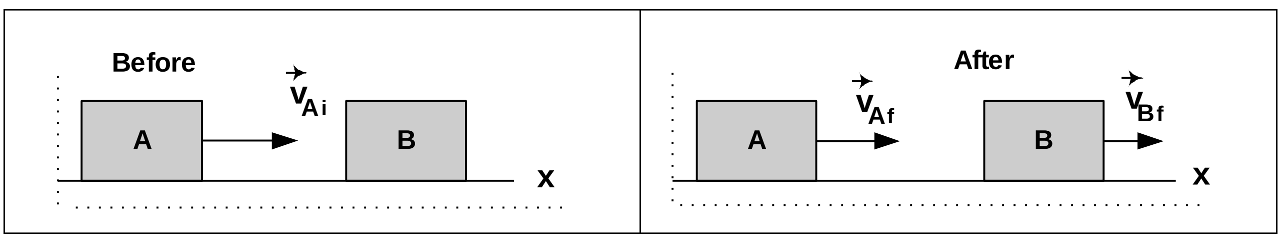
The final KE is: $KE_f\ =\ (1/2)\ m_1\ (v_{1f})^2\ +\ (1/2)\ m_2\ (v_{2f})^2$. This gives:

$$=\ (1/2)(2\ m_2)(30)^2\ +(1/2)\ m_2\ (20)^2\ =\ (1/2)(1800+400)\ m_2\ =\ 1100\ m_2$$

Since KE_f is not equal to KE_I, the collision is <u>inelastic.</u>

3) A block of mass 200 g, sliding with a speed of 12 cm/sec on a smooth level surface, makes a head-on, elastic collision with a block of unknown mass, initially at rest. After the collision the velocity of the 200 g block is 4 cm/sec in the same direction as its initial velocity. Determine the mass of the 2nd block and its speed after the collision.

<u>Solution:</u> We have a collision problem in 1-dimension. We draw both 'before' and 'after' pictures and select a coordinate system as shown.



Since the surface is frictionless, and since no work is performed by either mg or the normal, then the net force acting on the system is 0, and we have conservation of linear momentum:

$$\mathbf{p_{1I}} \oplus \mathbf{p_{2I}} = \mathbf{p_{1I}} \oplus \mathbf{p_{1f}}$$

Thus adding the x-components we have: $m_1v_{1I} +\ 0\ =\ m_1\ v_{1f}\ +\ m_2\ v_{2f}$.

This gives: $(200)(12) =\ (200)(4) +\ m_2\ v_{2f}$.

Since we have 2 unknowns we look for an 'energy condition'. We are told that the collision is <u>elastic</u>. Hence, we also have conservation of KE and write:

$$(1/2)m_1(v_{1I})^2\ +\ (1/2)m_2(v_{2I})^2\ =\ (1/2)m_1(v_{1f})^2\ +\ (1/2)m_2(v_{2f})^2$$

We may cancel the (1/2) factors, and we obtain: $(200)(12)^2 +\ 0\ =\ (200)(4)^2\ +\ m_2(v_{2f})^2$.

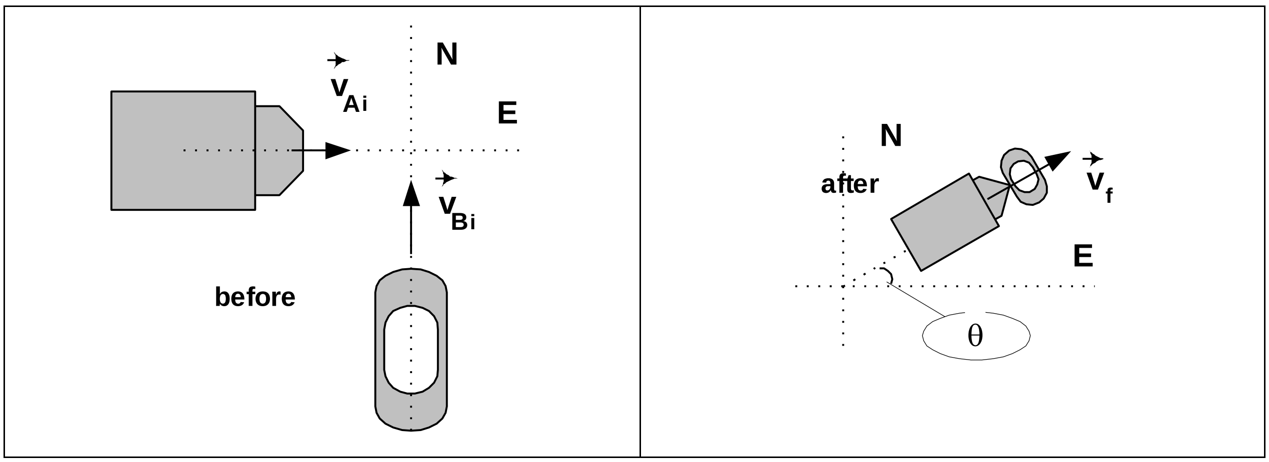
Thus $(m_2\ v_{2f})\ v_{2f}\ =\ (200)(8)\ v_{2f} =\ (200)(144) - (200)(16)$ or $8\ v_{2f} = 128$.

Hence: v_{2f} = 16 cm/sec, and m_2 = (200)(8)/(16) = 100 gm .

08-3

4) A 2000 kg automobile going east on Chestnut Street at 60 mi/hr collides with a 4000 kg truck which is going south across Chestnut Street at 20 mi/hr. If they become coupled on collision, what is the magnitude and direction of their velocity immediately after colliding?

<u>Solution:</u> We have a collision problem in 2-dimension. We draw both 'before' and 'after' pictures and select a coordinate system as shown. We have conservation of linear momentum: $\mathbf{p}_{Ai} \oplus \mathbf{p}_{Bi} = \mathbf{p}_{Af} \oplus \mathbf{p}_{Bf}$.



We have a completely inelastic collision problem. Applying Conservation of Linear Momentum we have:

$$m_1 \mathbf{v_{1i}} \oplus m_2 \mathbf{v_{2i}} = m_1 \mathbf{v_{1f}} \oplus m_2 \mathbf{v_{2f}}$$

We now rewrite this vector equation in terms of components (East & South).

<u>E comp:</u> $m_1 v_{1i} + 0 = (m_1 + m_2) v_f \cos\theta$

<u>S comp:</u> $0 + m_2 v_{2i} = (m_1 + m_2) v_f \sin\theta$

Thus we have 2 equations in 2 unknowns. (Note that since cons. of linear momentum is an <u>equi-dimensional</u> equation, we do not need to convert units).

<u>E comp:</u> $(2000)(60) = (6000) v_f \cos\theta$ <u>S comp:</u> $(4000)(20) = (6000) v_f \sin\theta$

Dividing the 'S' equation by the 'E' equation eliminates v_f and yields:

$$\tan \quad = (\sin\theta)/(\cos\theta) = (4000)(20)/(2000)(60) = 2/3 \rightarrow \theta = 33.7^o .$$

Then from the 'E' equation: v_f = (2000)(60)/(6000) cos 33.7 = 24.04 mi/hr .

5) A 0.144 kg baseball approaches a batter with a speed of 30 m/sec. The batter lines the ball directly

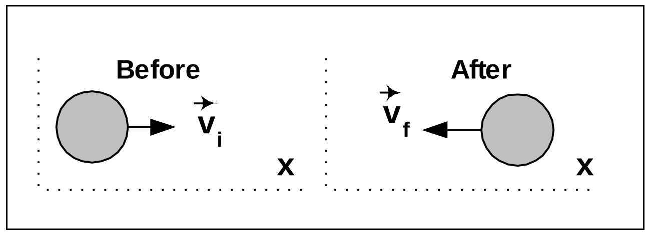
back to the pitcher with a speed of 40 m/sec. Find the change in momentum of the ball and the impulse exerted on the ball. If the bat and ball were in contact for 0.012 sec, find the average force exerted on the ball by the bat during this period.

08-4

<u>Solution:</u> We know very little about the nature of the force exerted on the ball by the bat. Thus rather than working from 'known force' to change in 'state of motion' we work backwards. We first determine the change in the linear momentum of the ball. The states of motion are specified by:

$\mathbf{p_i} = m\ \mathbf{v_i}$; $\mathbf{p_f} = m\ \mathbf{v_f}$. Thus the change in the linear momentum is: $\Delta\mathbf{p} = \mathbf{p_f}\ \Theta\ \mathbf{p_i} = m\,\mathbf{v_f}\ \Theta\ m\,\mathbf{v_i}$.

As always, we add (or subtract) vectors by the component method. We have selected a CS in the figure (x + to the right). Since there are no y-components then the y-component equation (0 = 0) gives us no information. The x-component equation is:



x-comp: $p_x = m\,v_{fx} - m\,v_{ix} = (.144)(-40) - (.144)(30) = -\ 10.08$ kg-m/sec.

That is, the magnitude of the change in momentum is 10.08 kg-m/sec and the <u>direction</u> is to the left! (negative value). From the 2nd law we have:

$$F_{net} = dp/dt \quad \rightarrow \qquad \boldsymbol{I} \equiv \int \vec{F}_{net}\, dt = \Delta\,\vec{p}$$

That is, the Impulse produced by the <u>net force</u> during any time interval is equal to the change in the linear momentum during that time interval. Since we neglect all forces in the problem other than the force of bat on the ball, then the net force is the force of the bat on the ball, and we have:

Impulse {bat on ball} = - 10.08 kg-m/sec .

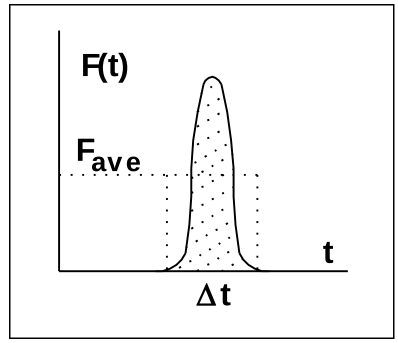
Again the negative indicates that the vector impulse ($\boldsymbol{I}$) is to the left (toward the pitcher). The graphical interpretation of impulse is that it's magnitude is the <u>area under the force curve</u> between the two times. In the figure we indicate what the force of the bat on the ball might look like. We have indicated that this force acts only for a brief time (t = 0.012 sec).

The impulse (10.08 kg-m/sec) is the area under this curve. We cannot determine the varying force F(t). However, we can calculate the <u>average force</u> which would produce this same impulse. F_{ave} would be the constant force which would have the same area over the same time period. That is:

$F_{ave} \quad \Delta t = 10.08$ kg-m/sec .

Hence: $F_{ave} = (10.08)_{/\ t} = (10.08)_{/(.012)} = 840$ N .

Thus the average force acting over 0.012 sec. is 840 N and points toward the pitcher.



08-5

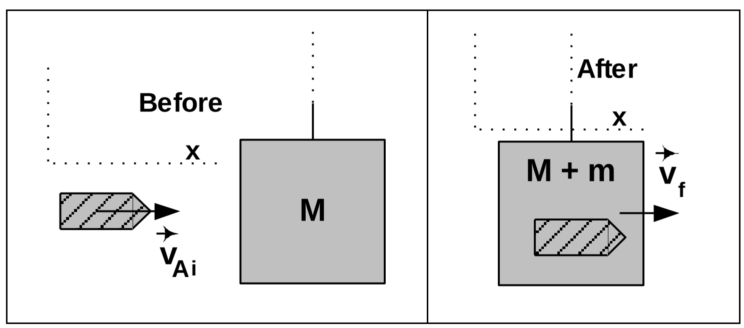
6) A 0.005 kg bullet going 300 m/sec strikes and is imbedded in a 1.995 kg block which is the bob of a ballistic pendulum. Find the speed at which the block and bullet leave the equilibrium position, and the height which the center of gravity of the bullet-block system reaches above the initial position of the center of gravity.

Solution: A 'ballistic pendulum' is a device which can be used to measure the muzzle velocity of a gun. The bullet is fired horizontally into the block of wood and becomes imbedded in the block. The block is attached to a light rod and can swing like a pendulum. After the collision the 'bob' swings upward and the maximum height it reaches is determined. From this information plus the masses of the bullet and block, one can determine the velocity of the bullet. The critical point to note in this problem is that we have two distinct problems:

Problem 1: A collision problem. Apply Conservation of Linear Momentum and energy relation. Problem 2: A work-energy type problem. Apply conservation of total mechanical energy.

Part 1: We draw before & after pictures, label the velocities, and choose a CS. Conservation of total linear momentum is:

$$m\,\mathbf{v_{Ai}} \oplus M\,\mathbf{v_{Bi}} = (m + M)\,\mathbf{v_f}\ .$$



In component form (for x-components) this gives:

$m\,v_{Aix} + M\,v_{Bix} = (m + M)\,v_{fx.}$ Since $v_{Bi} = 0$, we have:

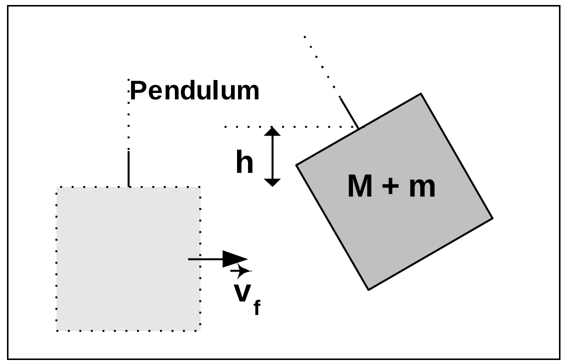
$$v_f = (m\,v_{Ai})\,/(m + M) = (.005)(300)/(2.00) = 0.75 \text{ m/sec.}$$

(Note: We were able to 'solve' the collision problem without an 'energy relation' since the collision was a perfectly inelastic collision. That is the two objects had the same final velocity. This condition is equivalent to an 'energy relation', since for such a collision the loss of KE is a maximum possible amount).

Part 2: In the work-energy part of the problem we note that the only force which performs work is gravity. Hence, we have only conservative forces present, and we have conservation of total mechanical energy.

08-6

We draw the figure indicating 'initial' and 'final' situations. We may choose the 0 level for gravitational potential energy anywhere we like. Hence, select $U_I \equiv 0$.



Then: $KE_I + U_I = KE_f + U_f$.

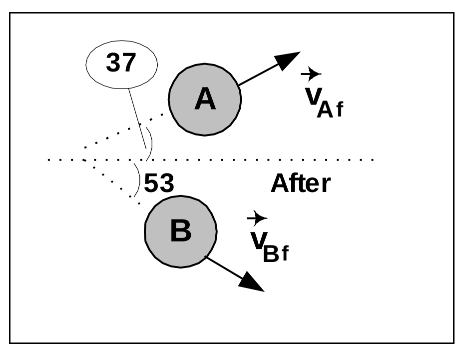
$$(1/2)(m + M)\, v_f^2 = 0 + (m + M)g\, h$$

Here v_f is the 'initial' velocity in this part of the problem (0.75 m/sec) and (m+M) is the combined mass of bullet & block.

Thus: $h = v_f^{\,2}/2g = (.75)^2/(2)(9.8) = .0287$ m or 2.87 cm.

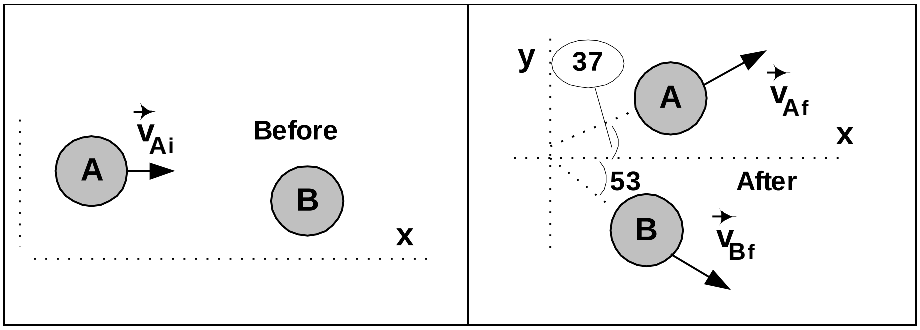
Note the reversal of this problem. If we know the masses and measure 'h', then from part 2 we can calculate 'v_f' (the initial velocity of bullet & block in the 2nd part of the problem). This is the same as v_f , the final velocity in the collision problem. Thus using this we can calculate v_{Ai} the 'muzzle' velocity of the bullet.

7) Object A traveling with a speed of 25 m/sec collides with an identical object B which was initially at rest. After the collision, the two objects are moving as shown. A) Determine the speeds of the two objects after collision. b) Was the collision elastic or inelastic? (show work).



Solution: We have a collision problem in 2-dimensions. We draw both 'before' and 'after' pictures and select a coordinate system as shown. We have conservation of linear momentum: $\mathbf{p}_{Ai} \oplus \mathbf{p}_{BI} = \mathbf{p}_{Af} \oplus$

p_{Bf} .



For the y-components we have: $0 = m_A v_{Af} \sin 37 - m_B v_{Bf} \sin 53 \rightarrow m v_{Af} (3/5) = m v_{Bf} (4/5)$

Hence $v_{Bf} = (.6/.8) v_{Af} = (3/4) v_{Af}$.

For the x-components we have:

$$m_A v_o = m_A v_{Af} \cos 37 + m_B v_{Bf} \cos 53 \rightarrow m v_o = m v_{Af} (4/5) + m v_{Bf} (3/5)$$

Canceling the 'm' in each term and using $v_o = 25$ m/sec.

$$25 = v_{Af} (4/5) + (3/4) v_{Af} (3/5) \text{ or } 25 = v_{Af}\{(4/5) + (9/20)\} = (25/20) v_{1f} . \text{ Thus}$$

$$v_{Af} = 20 \text{ m/sec} \quad \text{and} \quad v_{Bf} = 15 \text{ m/sec.}$$

Note that the problem did not say whether the collision was elastic or inelastic. Since we have both final velocities then we can directly answer whether or not the collision was elastic. The initial KE is simply

$$KE_i = (1/2) m v_o^2 = (1/2) m (25)^2 = (1/2)(625) m$$

The final KE would be:

$$KE_f = (1/2) m_A (v_{Af})^2 + (1/2) m_B (v_{Bf})^2 . \text{ This gives:}$$

$$= (1/2) m (20)^2 + (1/2) m (15)^2 = (1/2) m \{ 400 + 225\} = (1/2) m (625) .$$

The collision is elastic.