Solving Momentum Problems

Momentum:

For lack of a better definition, momentum is a measure of the "oomph" that an object has due to its motion. The more mass an object has and the more speed it has the more momentum it has. The formula for momentum is simply:

$$p=mv$$

Where p is momentum, m is mass, and v is velocity

Note that momentum is a vector quantity, so it is possible to have negative momentum. Any object that is moving in the direction opposite that defined as positive will have a negative momentum. You can also break a momentum vector into components or resolve momentum vectors into a single resultant.

Momentum is a conserved quantity. The momentum of a system will not change unless an outside impulse is applied to it. If the system remains isolated, its total momentum will not change. That does not mean that individual parts of a system cannot interact with each other and exchange momentums. Conservation of Momentum is a basic physics principle that allows us to solve many interesting problems.

The unit of momentum is a kg•m/s

Impulse:

The only way to change momentum is through impulse. Impulse is an outside force applied for a specific time. Obviously the harder you push and the longer you push the more the momentum will be changed. There is no specific variable for impulse, so we rely on its function, changing momentum, to provide a variable: Δp. The formula for impulse is:

$$\Delta p=Ft$$

Where Δp is impulse, F is force, and t is time.

Since both momentum and impulse are vector quantities, application of impulse can increase or decrease the momentum of a system. If the force is applied in the direction of motion, the momentum is increased, if opposite the motion, momentum is decreased. If the force is at an angle to the direction of motion it will change the net momentum in two or three dimensions.

Collisions and Explosions:

The main application of momentum techniques is in the solution of problems involving collisions and explosion. Collisions are when two or more objects run into each other. They can either stick together or spring back apart. Explosion are when two or more objects are pushed apart by an internal force. The "explosive" force can be provided by an actual explosion, a spring, a pair of magnets, etc.

Elastic and Inelastic Collisions:

An Elastic collision is one in which there is no permanent deformation. Good examples of elastic collision is a billiard ball colliding with another or a mass cart bumping into another with a spring in between. In an elastic collision both energy and momentum are conserved.

An Inelastic collision is one in which the objects stick together. Good examples of inelastic collisions are a ball of putty hitting and sticking to another ball or two railroad cars colliding and coupling together. In an inelastic collision momentum is conserved, but energy is not.

It is possible to have a partially elastic collision, where there is some deformation, but where the objects do not stick to each other. An automobile collision is a good example of this. Mathematically, a partially elastic collision is handled the same way as an elastic collision except that energy is not conserved.

Note that momentum is conserved regardless of the type of collision/explosion. That's why if you get a problem involving collisions or explosions you will most likely use momentum to solve it!

The momentum conservation equation for two masses:

$$m_1v_1+m_2v_2=m_1v_1'+ m_2v_2'$$

The ' character is pronounced "prime" and denotes the situation after the interaction. Each object has a velocity before and a velocity after the collision, but their masses remain the same.

Sample problem, elastic or partially-elastic collision:

Before we start with numerical examples, let's do a few simple thought experiments.

First, consider a ping-pong ball striking a pool ball. The collision is elastic because nothing permanently deforms. If you think about it, the ping-pong ball will rebound in the opposite direction while the pool ball will start moving slowly in the original direction of the ping-pong ball.

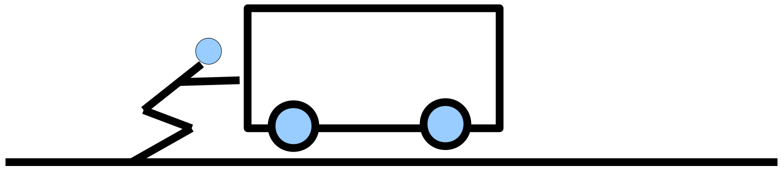
Second, consider the collision between two air-hockey pucks on a frictionless surface. If the path of the first puck is pointed directly throught the center of the second puck the first puck will stop and the second one will continue at the same rate. Again, there is no permanent deformation so the collision is elastic and it's pretty easy to visualize that the second puck will continue with the same velocity and direction as the first.

Third, consider the collision between a basketball and a volleyball. Again the collision is elastic and if the path of the first ball is through the center of the second ball the second ball will continue on the path of the first ball. In this case, the first ball will keep some of its forward velocity.

Next, consider inelastic collisions. In any case where the objects stick together they will both have the same final velocity and will end up sharing the momentum of the first object. This isn't nearly as exciting as the elastic case, but it is somewhat easier to handle mathematically!

Computation Examples

Let's start with a simple problem. In the movie *Little Miss Sunshine*, A Volkswagen bus of mass 1200kg needs a push to get started. The entire Hoover family (except for Richard, who is driving) pushes on the the bus for 15s, by which time it has reached a speed of 10MPH or 4.5m/s. Neglecting friction, with what force did they push on the bus?



The bus starts at rest, so it had zero momentum. Any momentum it picked up came from the impulse of the Hoover family pushing on the bus.

Calculate the final momentum of the bus:

$$p=mv$$

$$p=1200(4.5)$$

$$p=5400\text{kg}\bullet\text{m/s}$$

Now, the momentum went from 0 to 5400, so the change in momentum is 5400

Calculate the force needed to produce this impulse in 15s:

$$\Delta p=ft$$

$$5400=f(15)$$

$$5400/15=f$$

$$360\text{N}=f$$

Notice that any time that a push is used to change the velocity of a single object you can simply set the impulse and momentum equations equal to each other:

$$p = \Delta p$$

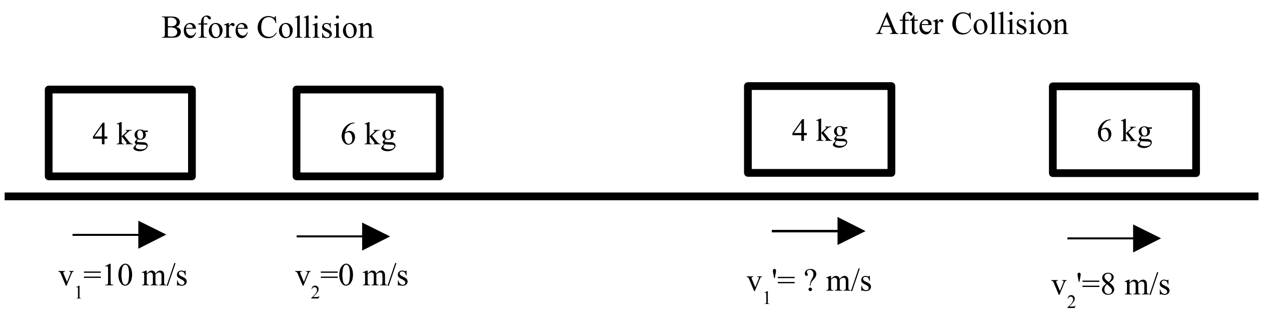
$$mv=ft$$

To be precise, v should be Δv, because the object might not be starting at rest, so the final equation for impulse on a single object is:

$$m\Delta v=ft$$

If the force direction is opposite that of the object (ie. a ball being caught), the force is negative.

Elastic Collision:
Let's take the case of a 4kg block with an initial velocity of 10m/s that is colliding with an 6kg block that is stationary. After the collision, the 6kg block is seen to be moving at 5m/s. Your job is to determine the velocity of the 4kg block after the collision. As you might imagine from the examples above, the light 4kg block is likely to rebound in the opposite direction after hitting the more massive 6kg block. Let's start with a diagram:



The computation is easy. We start with the collision equation:

$$m_1v_1+m_2v_2=m_1v_1'+ m_2v_2'$$

$$4(10)+6(0) = 4(v_1')+6(8)$$

$$40+0=4(v_1')+48$$

$$-8=4(v_1')$$

$$-2=v_1'$$

So, because the answer is negative, the 4kg mass will be rebounding at 2m/s in the direction opposite its original direction.

Is Energy Conserved?
Next, let's see if energy is conserved, as this is often the second part of collision problems (sometimes disguised as "how much energy was dissipated in the collision?"):

Kinetic energy before:

$$K = K_1+K_2 = ½m_1v_1^2+(1/2)m_2v_2^2 = ½(4)(10^2)+½(6)(0^2) = 200+0 = 200J$$

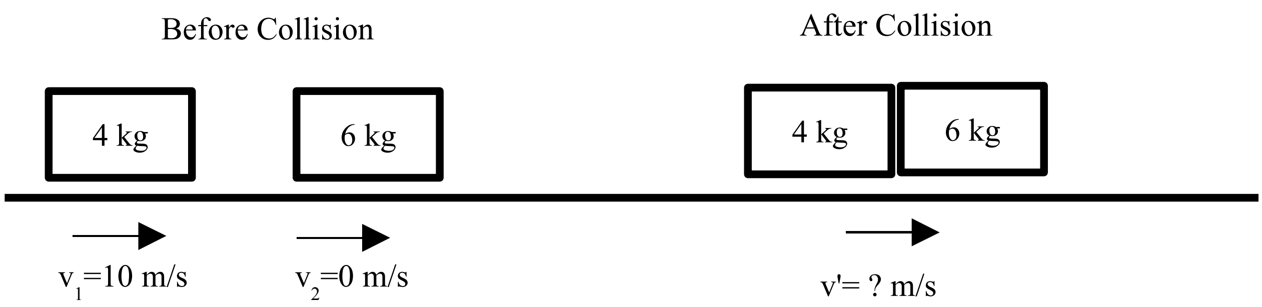
Kinetic energy after:

$$K = K_1+K_2 = ½m_1v_1'^2+(1/2)m_2v_2'^2 = ½(4)(-2^2)+½(6)(8^2) = 192+8 = 200J$$

Note that energy is NOT a vector, so the rebounding object has POSITIVE kinetic energy. Since the before and after kinetic energies are the same, we know that this is a perfectly elastic collision with no permanent deformation or heat released.

Now, for those who are very adept, you can find the velocities of both objects after an elastic collision even if you don't know the velocity of either object after the collision. To do this you have to write equations for momentum and energy conservation and solve them simultaneously. This is why the "Newton's cradle" toy only pops out the number of ball that are swung at it.

Inelastic Collision:
Let's take the case of a 4kg block with an initial velocity of 10m/s that is colliding with an 6kg block that is stationary. After the collision, both blocks are stuck together and are moving together. Your job is to determine the velocity of the linked blocks after the collision. Let's start with a diagram:



Again, we start with the collision equation:

$$m_1v_1+m_2v_2=m_1v_1'+ m_2v_2'$$

Note that after the collision v_1' and v_2' will be the same, since the blocks move together. You can rearrange the equation thus:

$$m_1v_1+m_2v_2=(m_1+ m_2)v'$$

$$4(10)+6(0) = (4+6)(v')$$

$$40+0=10v'$$

$$4=v'$$

So, the combined masses will be moving at at 4m/s in the original direction of the first mass.

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Kinetic energy after:

$$K = K_1+K_2 = ½m_1v_1'^2+(1/2)m_2v_2'^2 = ½(4)(4^2)+½(6)(4^2) = 32+48 = 80J$$

In this case there was a big reduction in kinetic energy after the collision, so we know that the difference in energies (120J) had to be lost either through doing work by permanently deforming one or both objects or as some sort of radiated energy such as heat or sound.