Laws of Nature and Theoretical Models

Chapter 4

Models

- This is important as engineers need to be confident that their designs can meet specifications.

- A poor design (bridge, building, airplane, could lead to loss of life.

- Engineers use models to predict the behavior of their designs.

Theoretical and Empirical Models

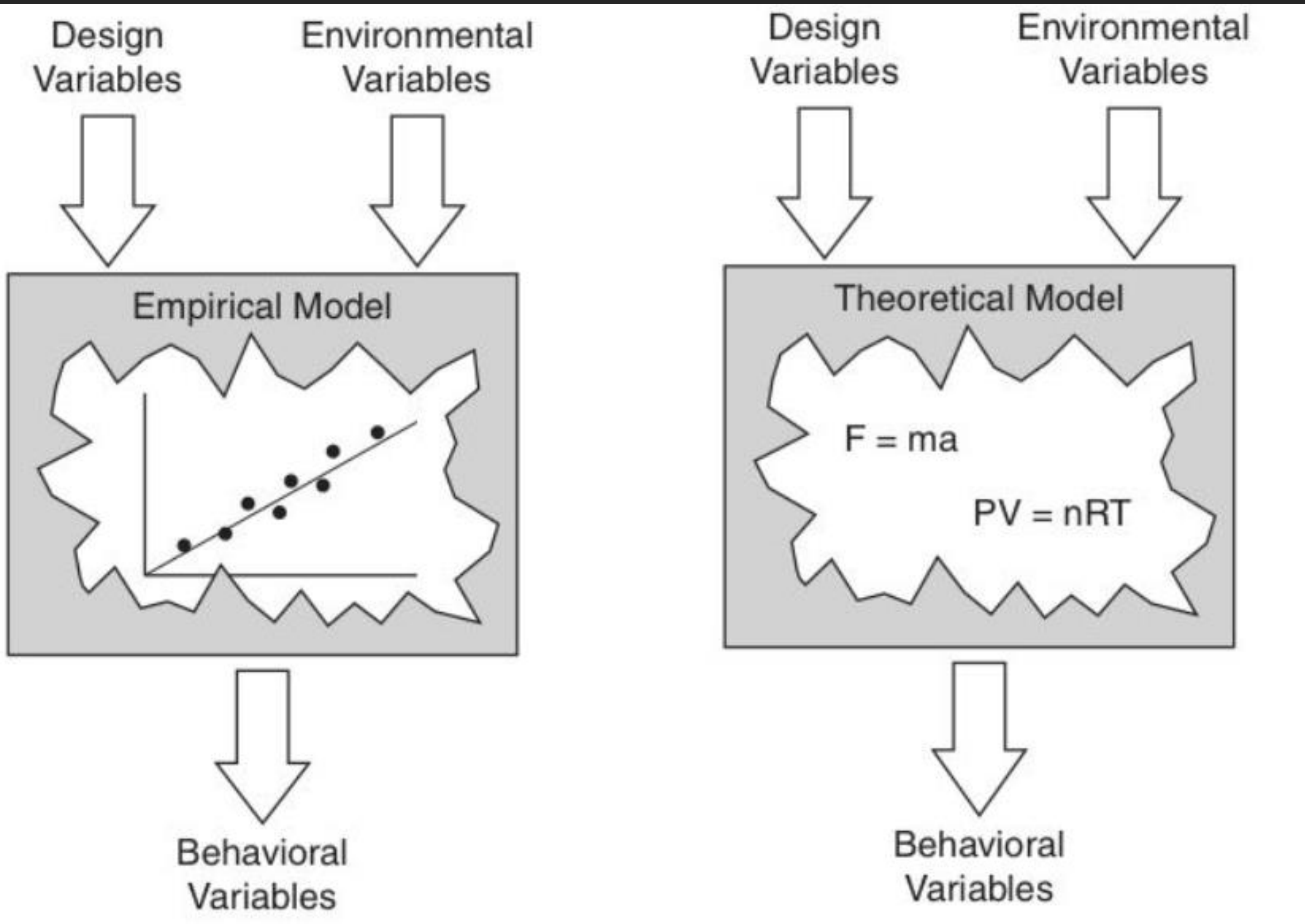

Figure 4.1 Mathematical model

Evolution of Theory

Consider the following three problems:

- • A 10 kg block is held at a height of 5 m. What is its potential energy?
-
- A 1 liter cylinder contains air at 100◦ C and at atmospheric pressure. What would be the pressure inside the tank if the cylinder were cooled to 0◦ C?
- A steel spring stretches by 2 cm when a 20 N weight is suspended from it. What would be the total amount of stretch if an additional 30 N were suspended from the spring?

Evolution of Models

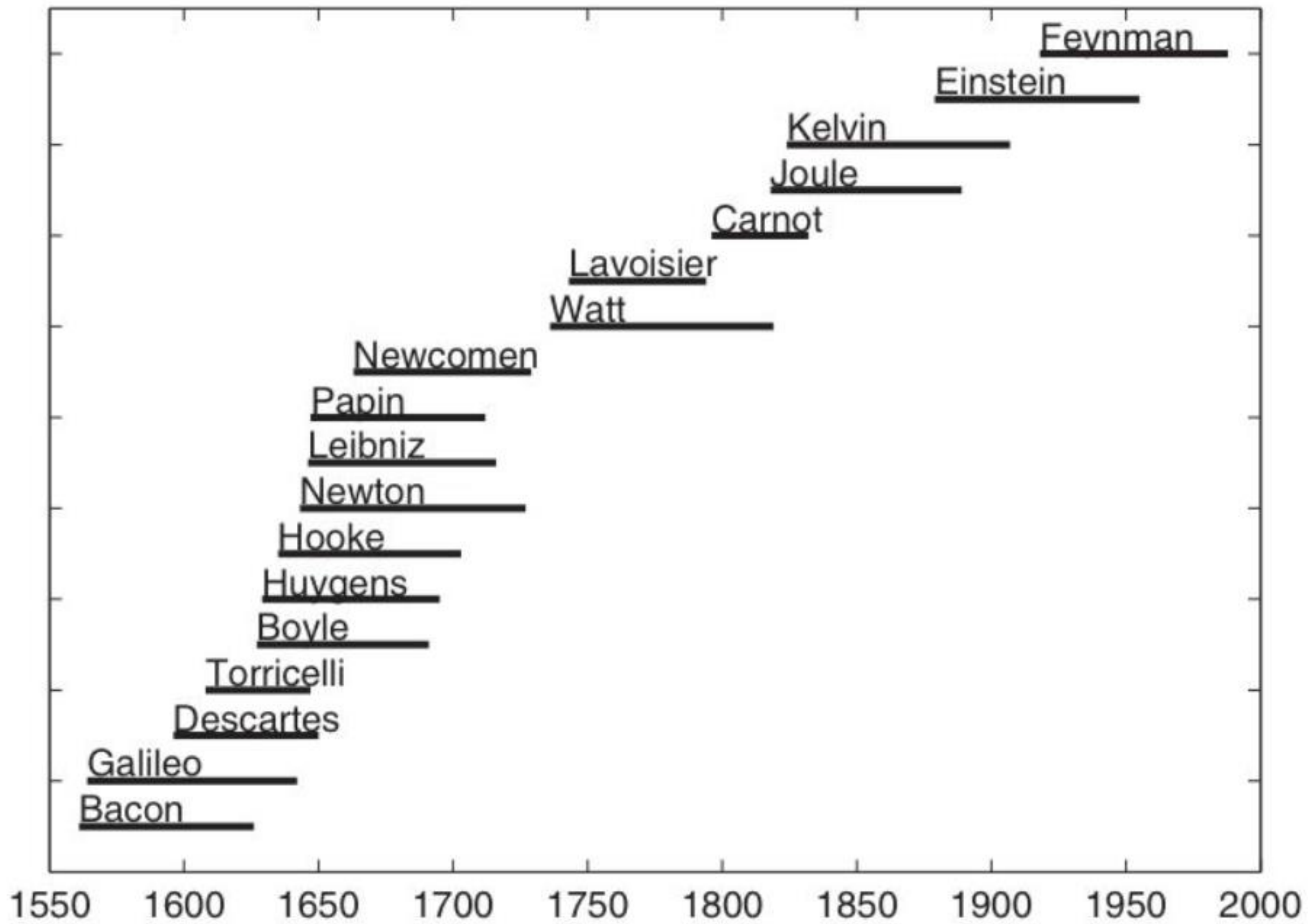

Figure 4.4 Lifetimes of scientists and engineers mentioned in this chapter.

Aristotle

- Heavier things fall faster.
- Speed of an object is inversely proportional to the density through which it falls.
- For violent motion, the speed of a moving object is directly proportional to the applied force.

Galileo

- Found that for a pendulum the square of the speed at the bottom of the pendulum was always proportional to the height from which it was released.

$$h = \text{constant} \times v^2$$

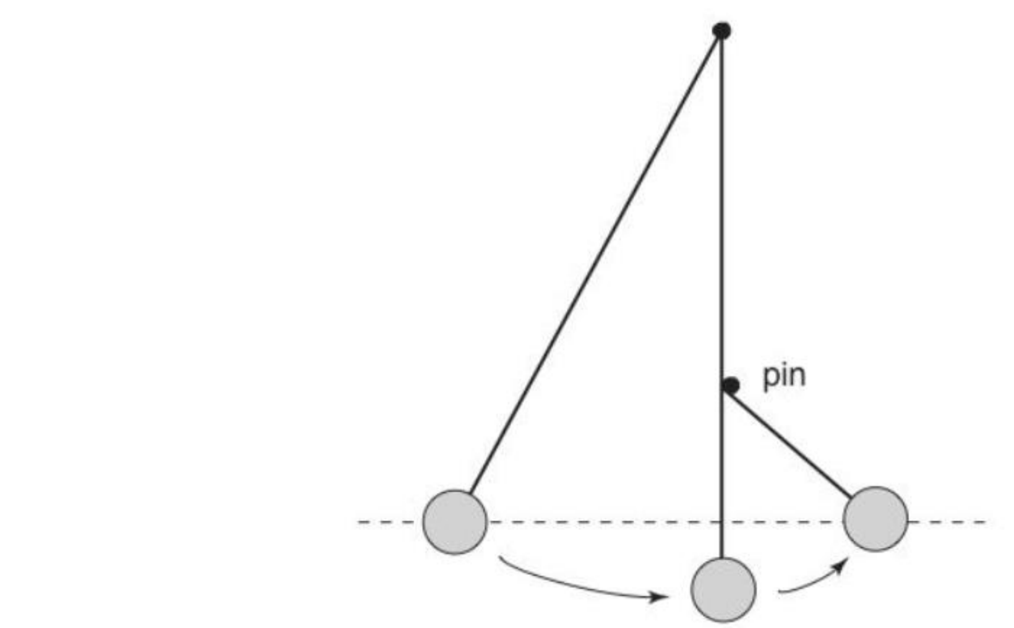

Figure 4.5 Galileo's interrupted pendulum.

Descartes

Descartes' Law of Conservation of Motion: That God is the primary cause of motion; and that He always maintains an equal quantity of it in the universe [Des91].

1. Each thing, insofar as in it lies, always perseveres in the same state, and when once moved, always continues to move.
2. Every motion in itself is in a straight line, and therefore things which are moved circularly always tend to recede from the center of the circle which they describe.
3. If a moving body A collides with moving body B and if A has less force to continue in a straight line than B has to resist it, A will deflect in the opposite direction and, retaining its own motion, will lose only the direction of its motion. If, however, A has a greater force than B does, then body A will move with body B and give it as much of its motion as it loses.

Descartes

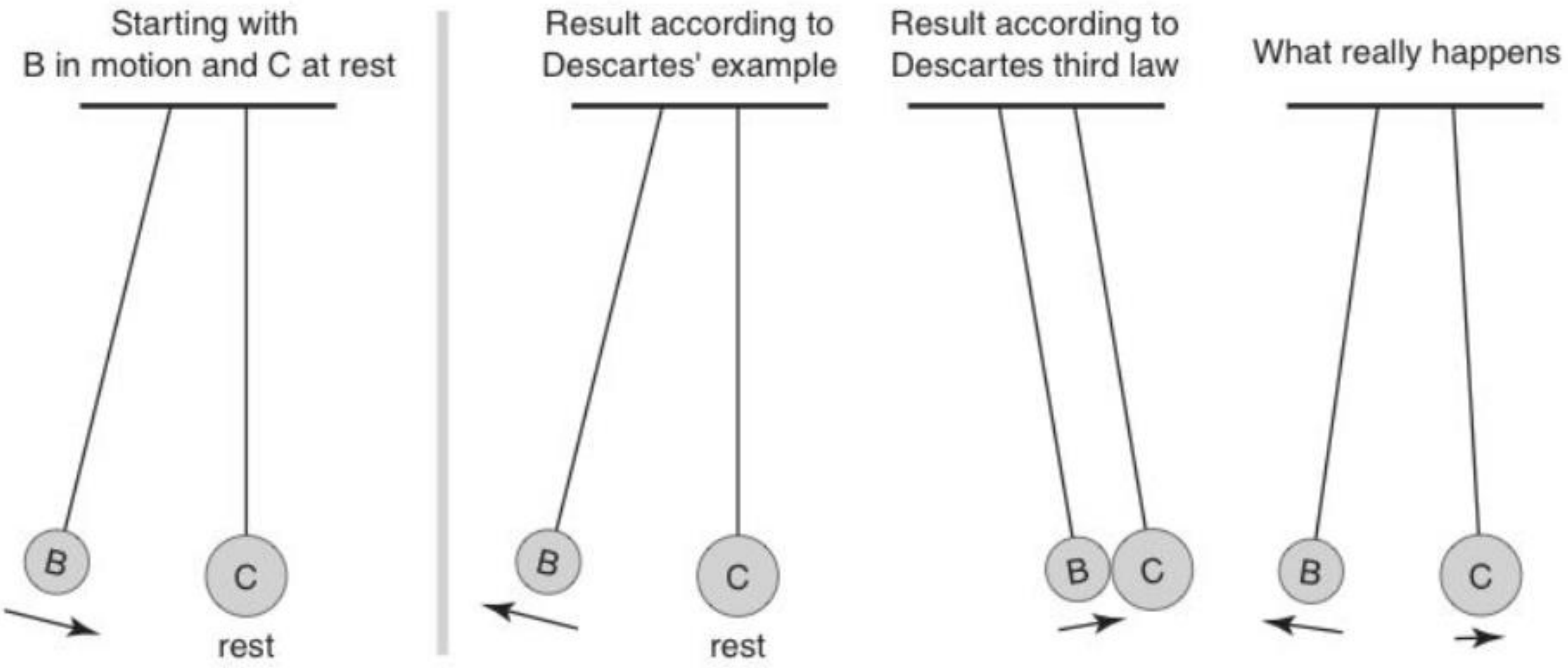

Figure 4.6 Descartes' Laws of Motion and examples in his *Principia Philosophiae* were wrong about collisions.

Huygens and Torricelli

Principle of relative velocity: $v_{1_i} - v_{2_i} = v_{1_f} - v_{2_f}$

Here there is 1 equation and 2 unknowns, so another law was called for

$$\frac{W_1}{W_2} = \frac{d_2}{d_1}$$

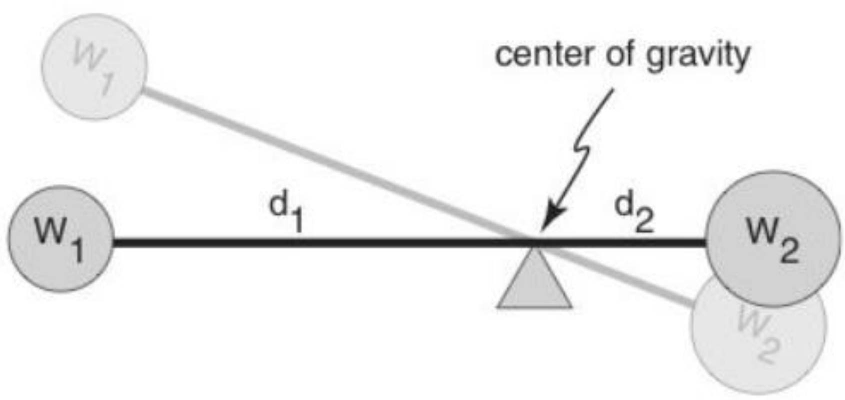

Figure 4.7 Center of gravity.

Huygens and Torricelli

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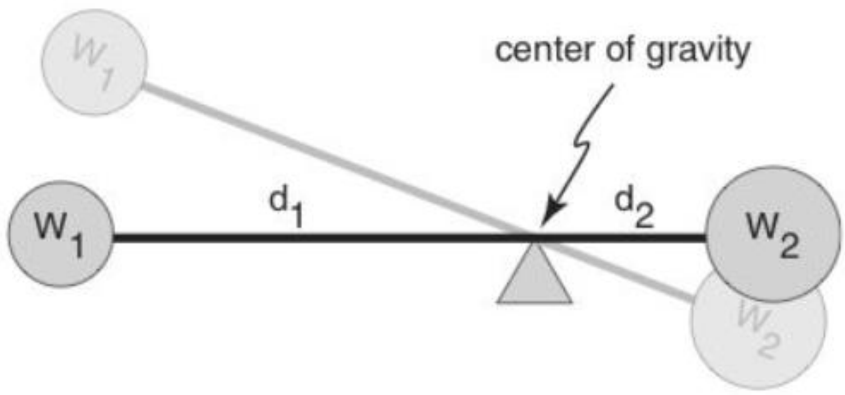

Figure 4.7 Center of gravity.

Huygens and Torricelli

Huygens determined this principle would work for pendulums dropped from differing heights

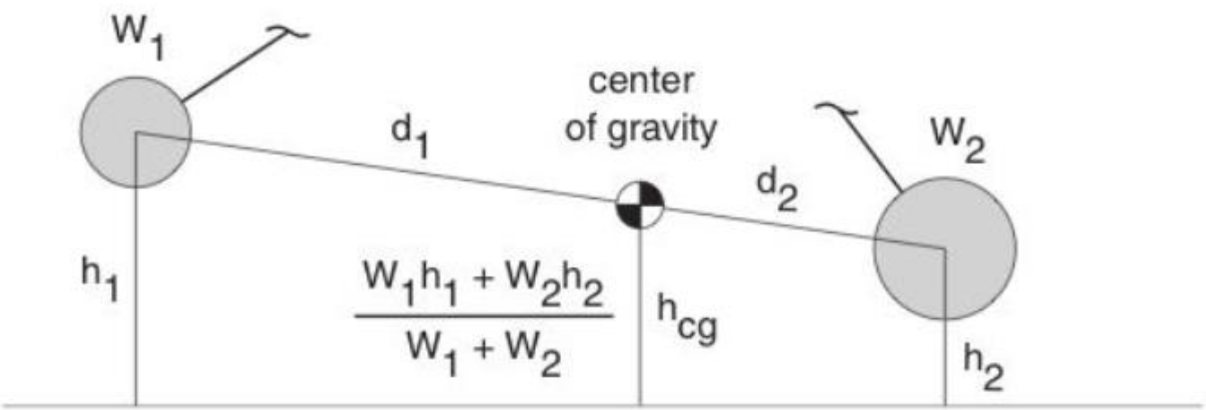

Figure 4.8 Calculation of the height of the center of gravity between two objects.

$$h_{cg} = \frac{W_1 h_1 + W_2 h_2}{W_1 + W_2}$$

Huygens' conservation of mv^2: $m_1 v_{1_i}^2 + m_2 v_{2_i}^2 = m_1 v_{1_f}^2 + m_2 v_{2_f}^2$

Huygens and Torricelli

- By taking size to mean mass and speed to mean velocity, we get:

Conservation of momentum: $mv_{1_i} + mv_{2_i} = mv_{1_f} + mv_{2_f}$

- The law of conservation of momentum as we know it today

Newton’ Laws of Motion

- Everybodyatrestremainsatrest,andeverybodyinmotionremainsinmotion, unless it’s compelled to change that state by forces impressed upon it.
- The change in motion (momentum) of a body is always proportional to the force impressed upon it.
- For every action, there is an equal and opposite reaction, or the mutual forces of two bodies upon each other are always equal in magnitude and pointed in opposite directions.

Newton's Impulse

$$F = \frac{\text{change in momentum}}{\text{change in time}}$$

$$m_A v_{A_i}^2 + m_B v_{B_i}^2 = m_A v_{A_f}^2 + m_B v_{B_f}^2$$

Work and Energy

- Leibniz described the organization of the universe as vis viva “living force”
- This term was later renamed as energy by Thomas Young.
- The term work was coined in the early 1800’s by Gaspard Coriolis.
- We still use this today W= F.s and is equivalent to the kinetic energy $1/2MV^2$

Work

- A curious thing about work is that the amount of energy moving a small force over a long distance or a large force over a short distance can be the same.

- Let’s consider a lever as an example of this.

Work

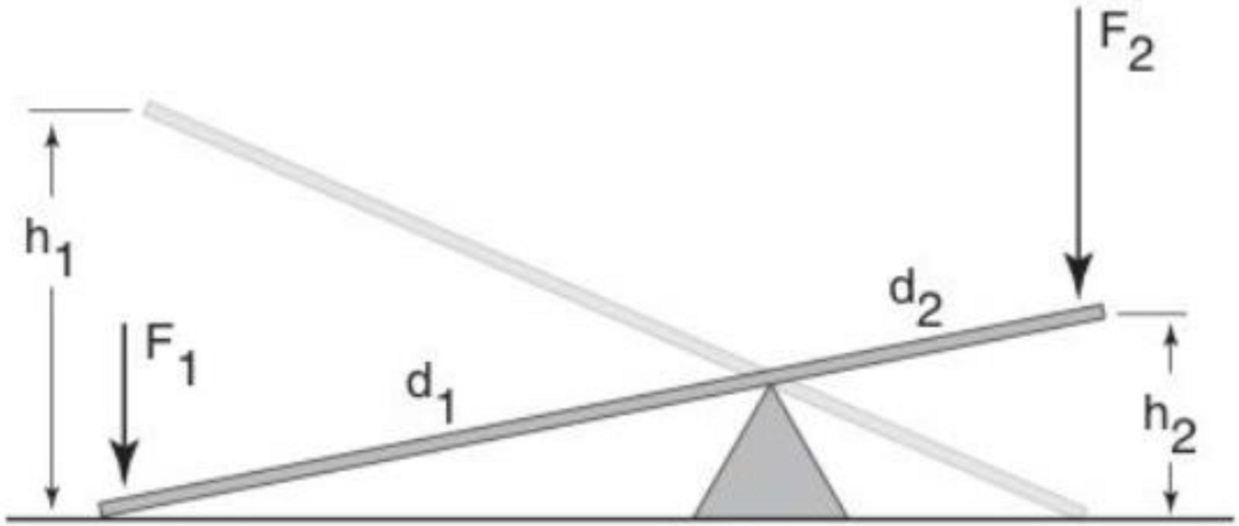

Figure 4.11 A lever.

$$\frac{F_1}{F_2} = \frac{d_2}{d_1}$$

By similar triangles,

$$\frac{d_2}{d_1} = \frac{h_2}{h_1},$$

and so

$$F_1 \cdot h_1 = F_2 \cdot h_2$$

Similarly

$$PV = nRT,$$

where

- P is pressure in Pascals (Pa),
- V is volume in m^3,
- n is the amount of gas in moles (mol),
- T is the temperature in degrees K,
- R is the gas constant, $8.31\ m^3 \cdot Pa \cdot K^{-1} \cdot mol^{-1}$.

Similarly

$$F = kx,$$

in which

- F = the force on the spring
- x = the extension
- k = the spring constant

$$\text{power} = \frac{\text{work}}{\text{time}} = \frac{\text{force} \times \text{distance}}{\text{time}}$$