Chapter 3
Understanding and Problem Solving

Philosophy

- Problem solving is a fundamental survival skill
- The reason we learn is to be able to solve problems
- To learn something deeply and meaningfully, you need to practice solving problems
- Problem solving comes naturally!!!
  - We even use it for entertainment

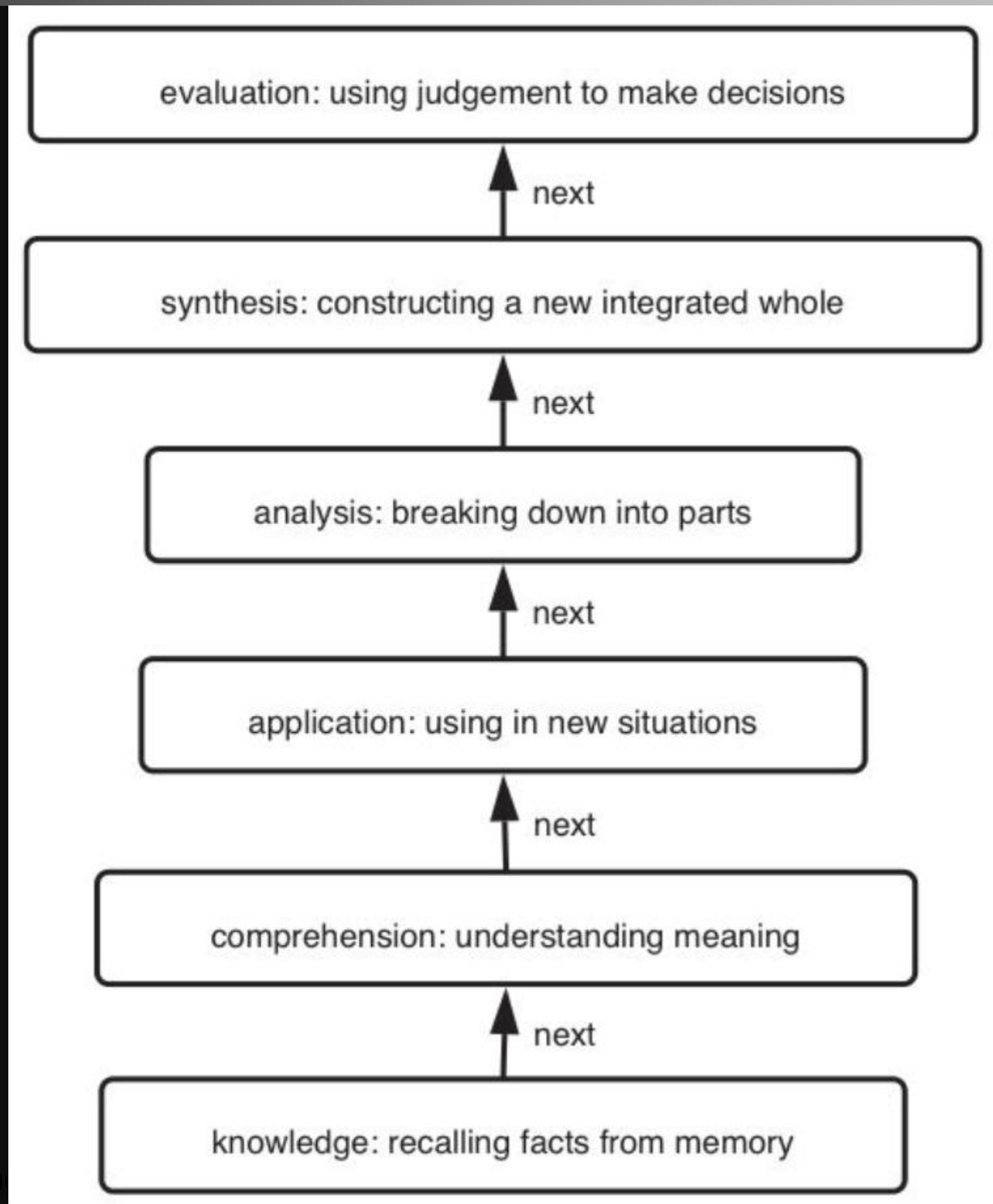
What Do You Know?

- Bloom's Taxonomy: a system for measuring levels of understanding
  - 6 levels, low to high:
    - knowledge, comprehension, application, analysis, synthesis, evaluation
- Why?

we will be offering Public Policy again in the Spring of 2014.

  - Used by (some) professors to design homework and test problems
    - What is the professor trying to get at with this problem?
  - For <u>you</u> to monitor <u>your own</u> understanding of a subject
    - A "lifelong learning" skill

Blooms Taxonomy



Knowledge: Recalling Facts

- Can recall, but not necessarily explain
- Keywords:

who	what	when	where

- Examples:
  - What is the value of π?
  - State the Pythagorean theorem.
  - A liquid has a pH of 3.2. Is it an acid or a base?

Comprehension: Understanding Meaning

- Able to explain or translate from one form to another.
  - e.g. translating a word problem to an equation
- Keywords:

summarize	explain	state in your own words
paraphrase	classify	give an example of

- Examples:
  - The three sides of a triangle have lengths a, b, and c. If $a^2 + b^2 – c^2 = 0$, which is the hypotenuse?
  - Give an engineering example that illustrates the expression, "an ounce of prevention is worth a pound of cure."

Application: Using in New Situations

- Typically demonstrated by using a concept learned in class to solve a problem
- Presumes lower levels of understanding
- Keywords:

demonstrate	show that	find, given assumptions

- Examples:
  - Given a right triangle with a hypotenuse of length 10 and a width of 8, what is its height?
  - My car won't start, why?

Analysis: Breaking Down into Parts

- Understand the *relationships* between the parts
- A critical step in solving any complex problem
- Keywords:

break down	contrast	compare
decompose	why	what might cause

- Examples:
  - Why do most computers today represent information with 1s and 0s?
  - My circuit is supposed to produce the sum of two numbers but it doesn't. Why?
  - What are the main differences between a JPEG and GIF file?

Synthesis: Building a New Integrated Whole

- Not just putting pieces together, but doing so in an original way
- Follows from analysis: break a problem down before synthesizing a solution
- Keywords:

create	compose	design	devise
develop	prove	plan	formulate

Examples:

- **Develop a strategy for studying for a exam.**
- **Write an essay that either defends or disputes the use of genetic engineering in food production**

Evaluation: Using Judgment to Make Decisions

- More than just choosing—requires a synthesis and analysis of possible outcomes
- Using judgment to make decisions
- Keywords:

judge	optimize	which is best
decide	defend	what is most appropriate

- Examples:
  - A friend told you that he has a copy of tomorrow's exam. What should you do?
  - What is your next move in a game of chess?

Exercise: Which levels do these problems test?

1. What is a “bit”?
2. Convert the following binary number 1001011 to a decimal and hexadecimal number.
3. Convert the decimal number 73 to a binary number using both the subtraction-based and division-based methods.
4. How many bits are needed to represent the decimal number 1234 as a binary number?
5. (Vahid 2.10c) Convert the following English problem statement to a Boolean equation: “An irrigation system should open the sprinkler’s water valve if the system is enabled and neither rain nor freezing temperatures are detected.”

Exercise: Which levels do these problems test?

6. (Vahid 1.30) Suppose that decrypting a frame of video for a set-top box requires 3 tasks, A, B, and C. The execution times for each task on a microprocessor versus a custom circuit are 100 ms vs.1 ms for A, 10 ms vs. 2 ms for B, and 15 ms vs.1 ms for C. Partition the tasks between the microprocessor and custom circuitry to minimize the amount of custom circuitry, while meeting the constraint of decrypting at least 30 frames per second.
7. A half-adder is a digital circuit that takes two 1-bit numbers as input and produces a 2-bit number as output. Design a half adder using just AND, OR, and NOT gates. Hint: design this as two circuits, one that calculates the least significant bit of the output and another that calculates the most significant bit.

Getting Good results from Your Learning Efforts

- Get Ready to learn: be physically and mentally prepared.
- Take stock of what you know.
- Acknowledge misconceptions.
- Beware of information overload!
- Take notes on notes.
- Use concept maps.

A Framework for Problem Solving

- Can apply this pattern to problems of all sizes
  - In school and beyond
- Approach based on
  - Wankat (Chemical Engineering, Purdue) and Woods (McMaster University, Canada).
  - Suggested by Ed Maginn (Chemical Engineering, ND)
- 7+1 steps:

0. I can!	**3. Plan**	6. Generalize
1. Define	4. Implement	7. Present results
2. Explore	5. Check	

Step 0: I Can!

- Have a positive attitude before you start
- See problem solving as a challenge, not a drag

Step 1: Define

- Identify the “knowns”
- Identify the “unknowns”
- State in simpler terms
- Draw a picture

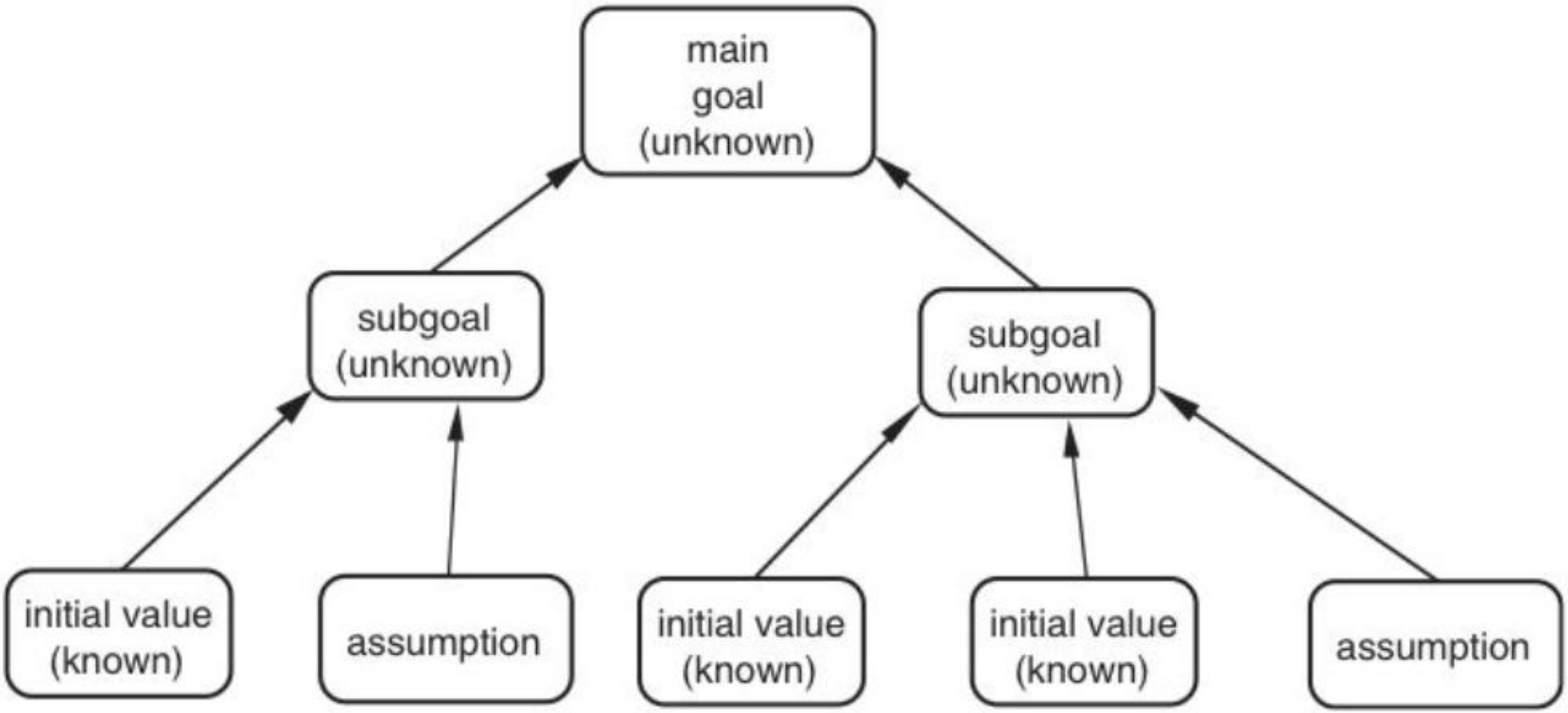
Step 2: Explore

- A pre-planning step—take stock of what you have and what you might need
- Does the problem make sense?
- Do you need to make any assumptions?
- What are the key concepts and possible approaches?
- What level of understanding is being tested?

Step 3: Plan

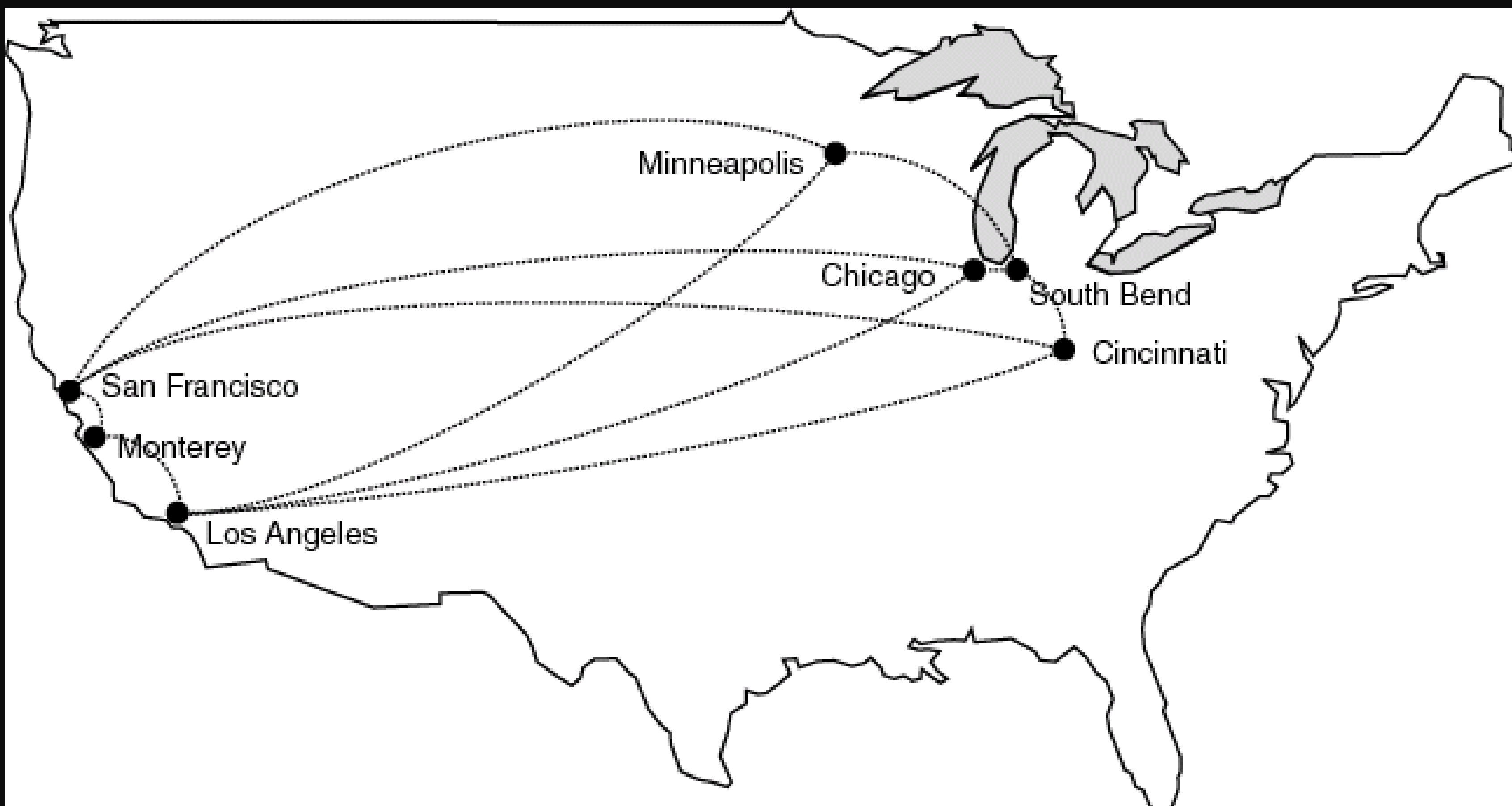
- Find a path between here (initial state) and there (goal)
- The most important step—novices often try to skip it with disastrous results
- The step where you’re most likely to get stuck
  - We’ll discuss strategies for getting “unstuck” soon
- Planning is messy
  - Lots of revision and backtracking
  - The neat solutions in many textbooks hide the tortuous path that the author might have followed
- Requires advanced level of understanding
  - Breaking down (analyzing) a problem
  - Formulating (synthesizing) alternative plans
  - Evaluating which path to take

Step 3: Plan

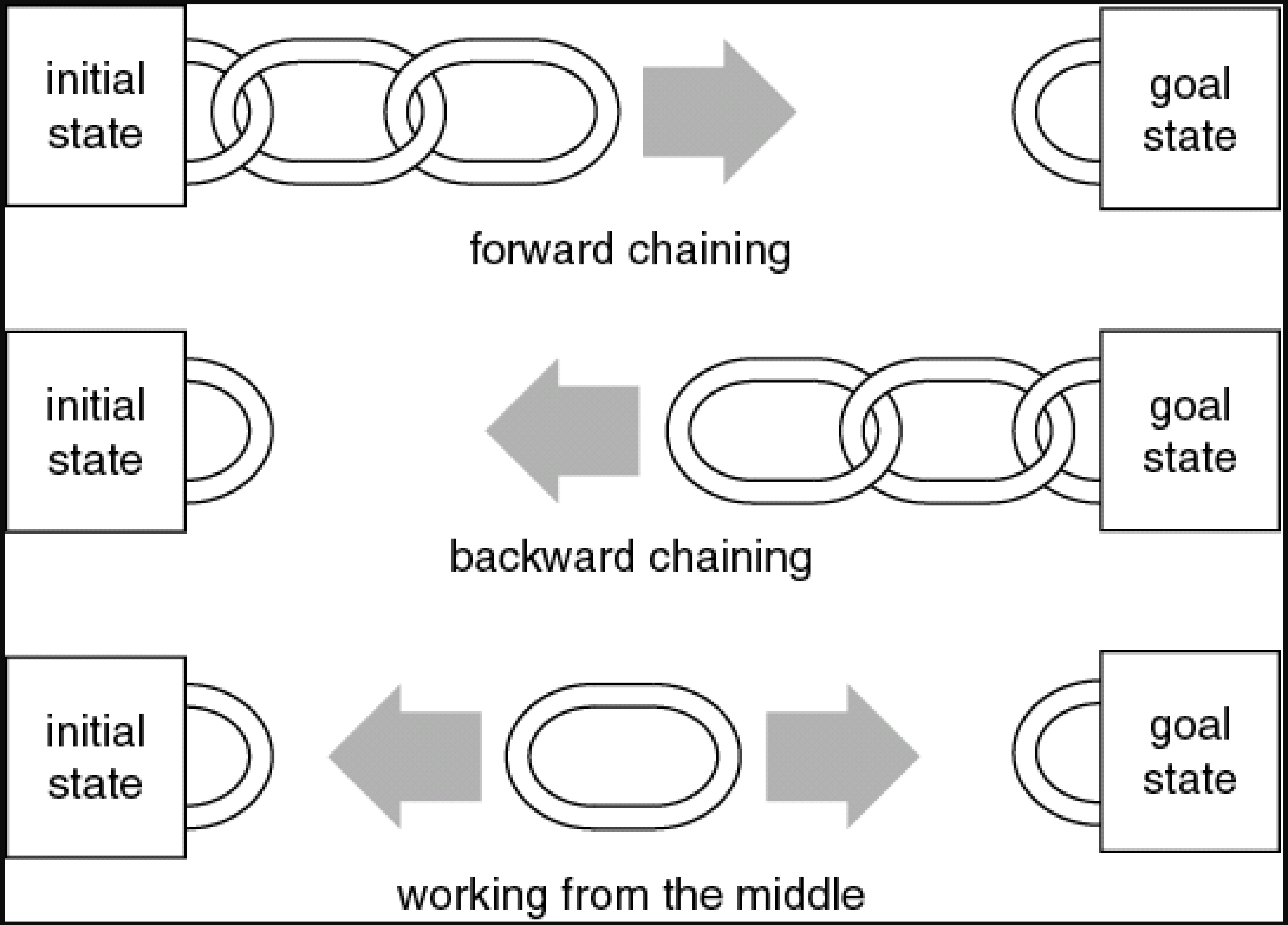

main goal (unknown)
subgoal (unknown)
subgoal (unknown)
initial value (known)
assumption
initial value (known)
initial value (known)
assumption

Planning a Path Between Here and There

- Booking flights from South Bend, IN to Monterey, CA
  - Work forward, work backward, or from the middle



Chaining Together a Solution



Step 3: Plan

- divide and conquer
- work forward
- work backward
- try a simpler problem
- take a break

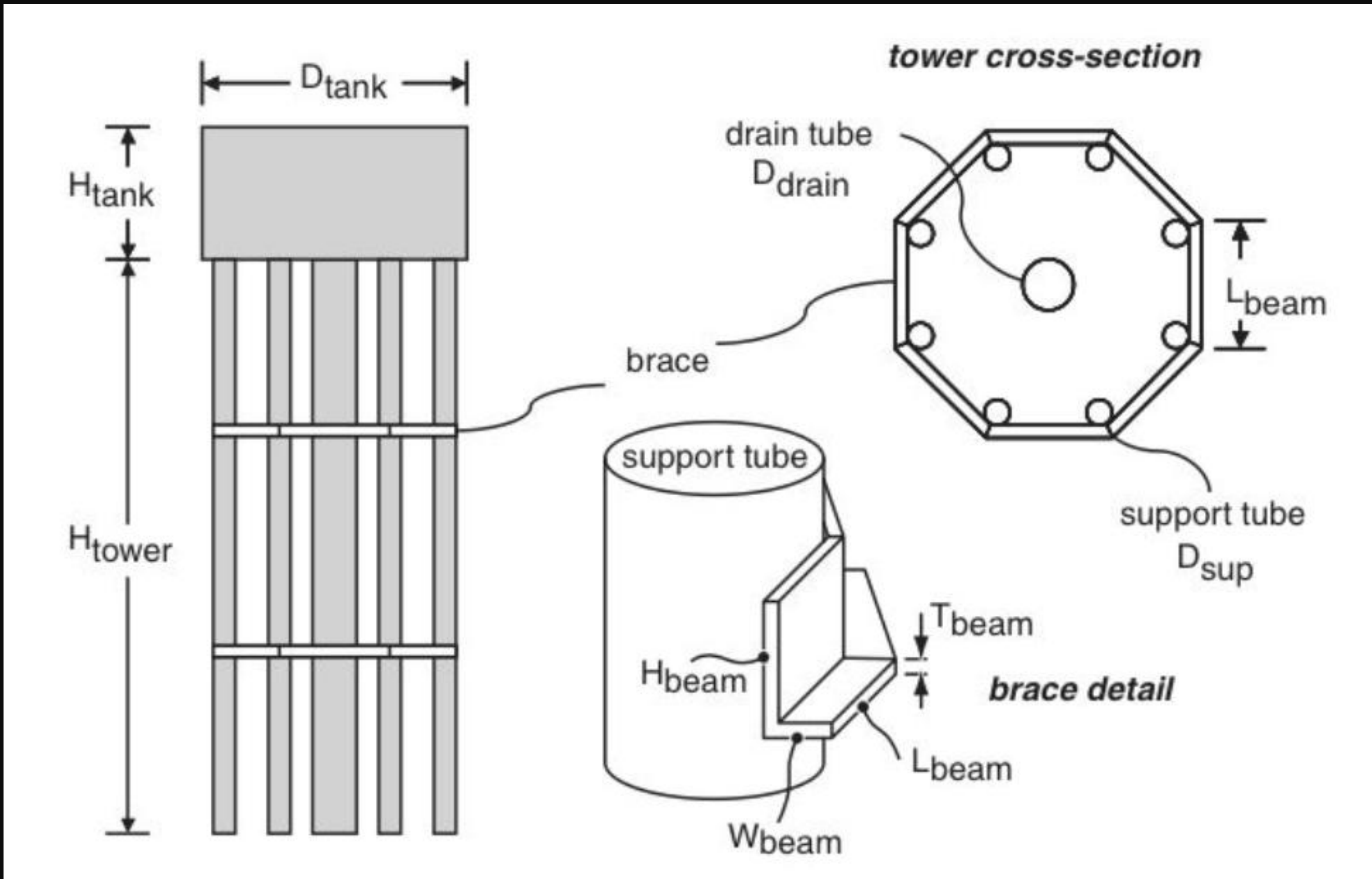
Step 3: Plan

Example 3.1

Painting a Water Tower

Develop a plan for determining how many gallons of paint would be needed to apply a single coat to the water tower shown in Figure 3.5. The water tower consists of a tank on top of a tower. The tank is a sealed cylinder. The tower consists of eight support tubes arranged symmetrically around a wider drain tube. Two octagonal braces made from L-beams (that is, beams shaped like the letter "L") surround the set of support tubes at levels one-third and two-thirds up the height of the tower.

Step 3: Plan



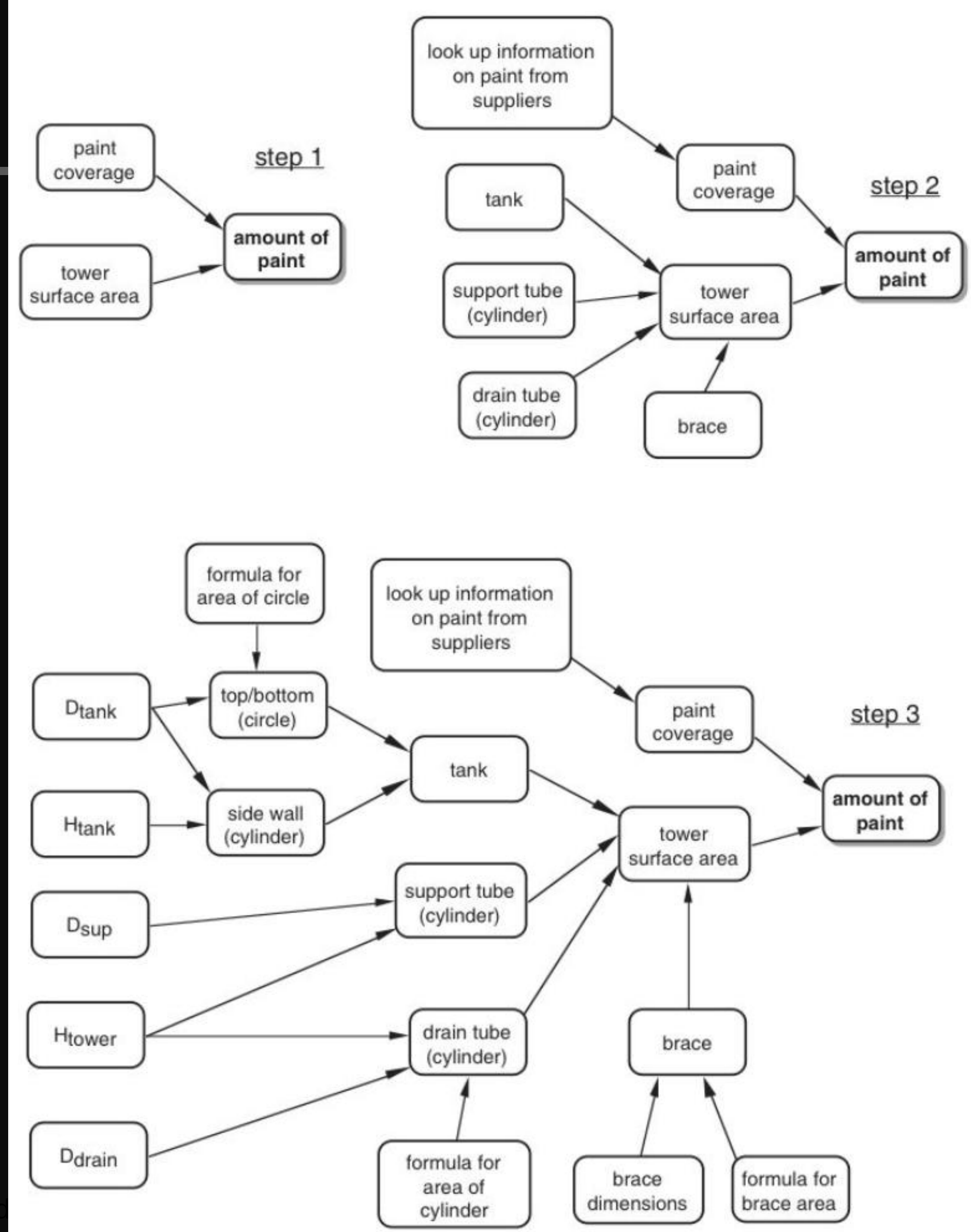
D_{tank}
H_{tank}
H_{tower}
tower cross-section
drain tube
D_{drain}
L_{beam}
brace
support tube
D_{sup}
support tube
T_{beam}
H_{beam}
brace detail
L_{beam}
W_{beam}

Step 3: Plan

Solution The initial state in this problem is that we know the dimensions of the water tower. The goal state is to know the quantity of paint required to cover the tower. In developing the plan, we'll mainly use two heuristics: "working backwards" from the goal toward the initial state, and "divide and conquer" to break the tower into component parts.

Figure 3.6 illustrates the development of the plan in three steps. The first step simply states that finding the amount of paint depends on the surface area of the water tower and the coverage of the paint, which is the number of gallons required per unit area. The second step of the plan continues to work backwards, breaking the tower down into component pieces, and determining that the paint coverage should be obtained by looking it up from a supplier. The final step adds the operations for calculating the areas of the parts of the tank from the given dimensions. It also identifies the geometric shapes of the components, and notes the formulas needed to calculate the areas.

Step 3: Plan



step 1
paint coverage
tower surface area
amount of paint
step 2
look up information on paint from suppliers
tank
support tube (cylinder)
drain tube (cylinder)
brace
paint coverage
tower surface area
amount of paint
step 3
formula for area of circle
look up information on paint from suppliers
Dtank
Htank
Dsup
Htower
Ddrain
top/bottom (circle)
side wall (cylinder)
tank
support tube (cylinder)
drain tube (cylinder)
formula for area of cylinder
paint coverage
tower surface area
amount of paint
brace
brace dimensions
formula for brace area

Step 4: Implement

- Begins *after* planning
- The step where you do the math
- Goes much more smoothly if you follow a plan
- Consider re-planning if implementation is not going smoothly

Step 5: Check

- Sanity check
  - Does the answer make sense? (negative time?)
  - Checking units can be a big help
- Test cases
  - Can you plug in values to get a known solution
  - Especially useful for checking digital circuits

Step 6: Generalize

- What did you learn from solving the problem?
  - Can you apply this to another situation?
- In retrospect, could you have solved it more efficiently?
- Are there any problems or bugs that you should remember in case you run into them again?

Step 7: Present the Results

- Show your work, but be concise
- Give good directions, be convincing
  - Get good grades in school and win contracts at work
- Be neat
- Suggested outline: simplification of framework
  - Given
  - Find
  - Diagram
  - Plan
  - Analysis (implementation)
  - Comments

Example

- How much CO_2 does a typical car produce per year?
  - Suggested by Ed Maginn, Chemical Engineering, ND

Getting Unstuck

- Heuristics: rules of thumb, learned by experience
  - Run through the list, alone or with a friend

- Restate in simpler terms
- Draw a picture
- Related problem?
- Work backwards/forwards
- Divide and conquer
- Unnecessary constraints?
- Discuss
- Try a scaled-down or simpler problem

- Guess and check
- Use an analogy
- Change perspective
- Do easy part first
- Plug in numbers
- Keep track of progress
- Change representation
- Listen to hunches
- Take a break . . .

Write it Down and State in Simpler Terms

- Example of multiplying a 4 and 2 digit number.
- Package related information in chunks
- Rewrite word problems symbolically or mathematically, if possible.

Draw a Picture

Example 3.2

Draw a Picture

Given the following description of a tic-tac-toe board, determine what X's next move should be.

> The first X is placed in the center of the board. The first O is placed one square to the left and one square up from the first X. The second X is placed in the square that is two squares to the right of first O. The second O is placed immediately below the first O.

Draw a Picture

Solution It's difficult to solve this problem without drawing a picture of the board. With a picture it's easy.

O		X
O	X	

Do you know a related problem?

Example 3.3

Related Problems

Which two of the following three problems have the most in common?

1. A right triangle has legs of lengths 3 m and 4 m. What is the length of its perimeter?
2. A rectangle has sides of lengths 3 m and 4 m. What is the length of its perimeter?
3. A person is rowing a boat across a river. If the person is rowing at a speed of 3 mph perpendicular to the bank and the river is flowing at a rate of 4 mph, what is the speed of the boat in the direction of travel?

Do you know a related problem?

Solution Although problems 1 and 2 both involve calculating the perimeters of polygons and problem 3 involves calculating the speed of a boat, the solutions to problems 1 and 3 both require using the Pythagorean theorem to calculate the hypotenuse of a right triangle. Having the experience of solving problem 2 would not help you solve problems 1 or 3, but having the experience of solving either of these would help you solve the other. Therefore, problems 1 and 3 have the most in common.

Work Backwards

Example 3.4

How Many Games in a Tournament?

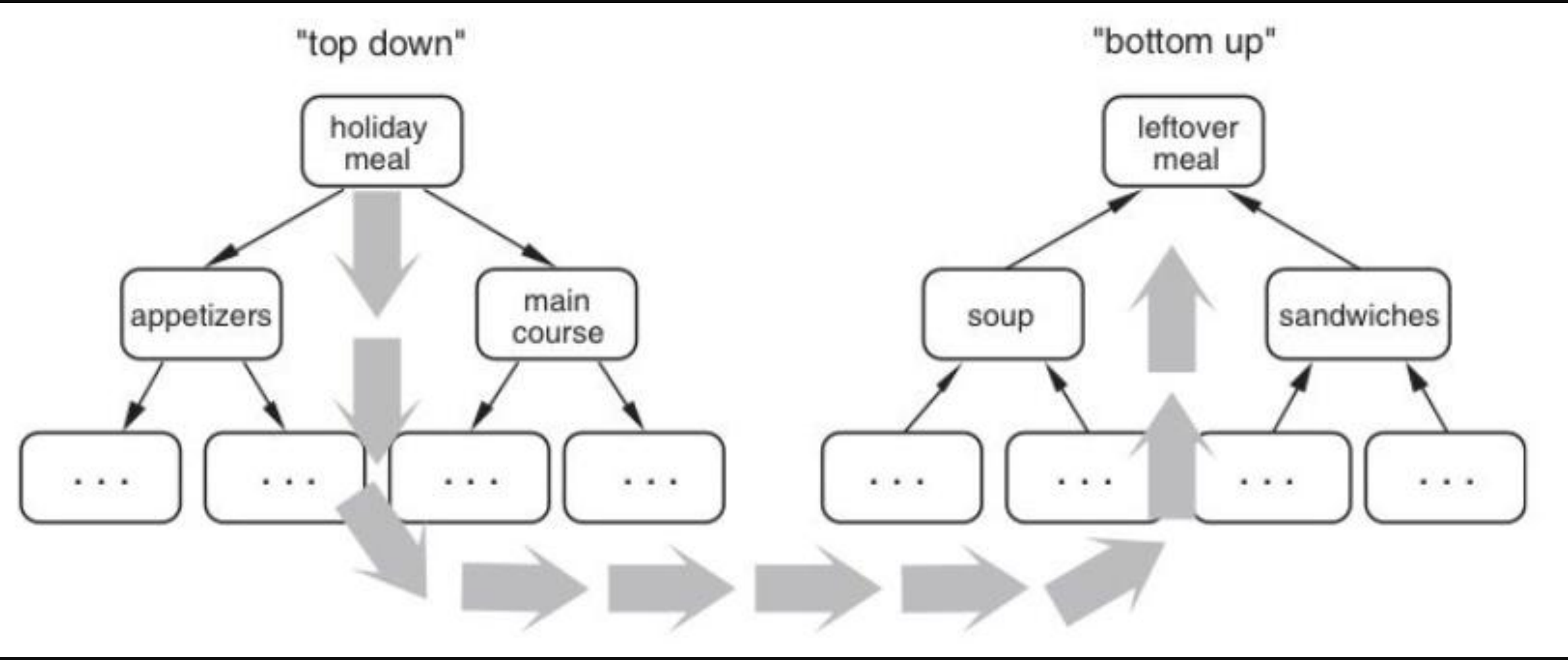
The NCAA college basketball tournament starts with 64 teams. If teams are eliminated after a single loss, how many total games are played in determining the winner?

Work Backwards

Solution The working-forward approach to solving this problem is to start with the 64 teams and systematically reduce them to 1 winner, adding up the number of games along the way, beginning with 32 games in the first round, 16 in the second round, and so on until you get to 1 game in the finals.

Before you add this up and get a result, consider working backwards from the goal. At the end of the tournament, there is 1 winner and 63 losers. Every game played produced 1 loser, so there must have been 63 games. Now go back and complete the forward-chaining approach and see that you get the same answer.

Top Down or Bottom Up



Divide and Conquer

Example 3.5

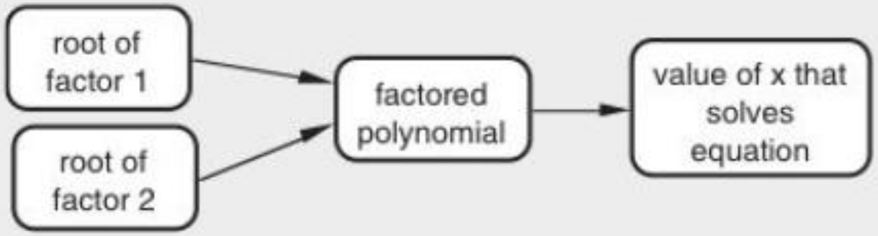
Divide and Conquer: Finding the Roots of a Polynomial

Find the values of x that solve the following equation:

$$x^2 - 7x + 12 = 0$$

Divide and Conquer

Solution Using a divide and conquer approach, we can factor the second-order polynomial into two first-order polynomials, and then find the values of x for which either of these equal zero. Because the factors are independent, we can find the roots separately.



$$(x - 3)(x - 4) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = \{3, 4\}$$

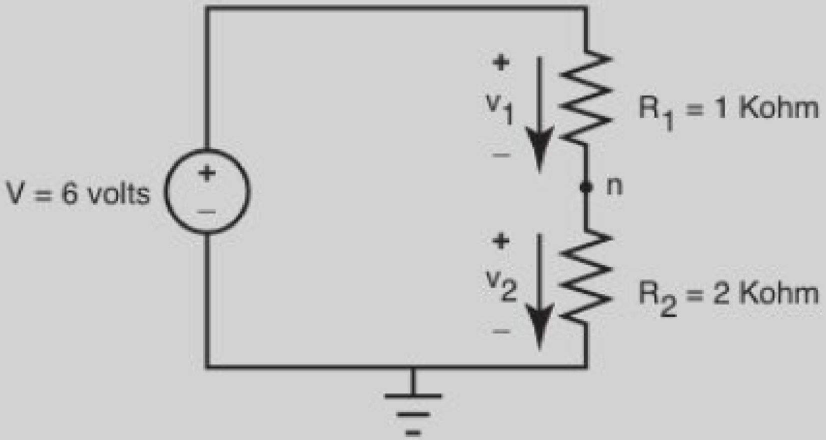
Divide and Conquer

Example 3.6

Voltages in an Electrical Circuit

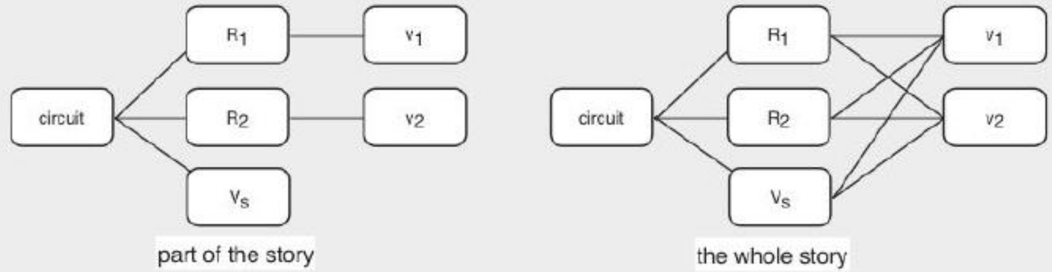
Determine the voltage drops v_1 and v_2 across the two resistors R_1 and R_2 in the circuit below. Consider the following laws in determining your solution.

- *Kirchhoff's Voltage Law:* The sum of the voltage drops (or negative voltage gains) in a loop equals zero.
- *Kirchhoff's Current Law:* The sum of the currents flowing into a node is zero.
- *Ohm's Law:* The current through a resistor equals the voltage across it divided by the resistance.



Divide and Conquer

Solution Perhaps thinking of other problems in which you were able to break a system down into separate components, such as the water tower problem in Example 3.1, you may be tempted to try to solve for v_1 in terms of R_1 and v_2 in terms of R_2 independently. The problem with this thinking, however, is that v_1 and v_2 each depend on *both* R_1 and R_2, as shown below:



This is because the voltage that drops across the resistors depends on the current flowing through the circuit, which flows through both resistors and therefore depends on both. Systems with mutual dependencies such as this circuit typically lead to a system of several equations in several unknowns that must be solved *simultaneously*. In this case, we could formulate two equations using Kirchhoff's voltage and current laws in terms of the unknown voltages v_1 and v_2. First, according to Kirchhoff's voltage law, the sum of the voltage drops across the resistors minus the voltage gain across the power supply equals zero:

$$v_1 + v_2 - V = 0$$

Using Kirchhoff's Current Law and applying Ohm's Law to determine the current through a resistor, the current flowing into node n through R_1 minus the current flowing out of n through R_2 equals zero:

$$\frac{v_1}{R_1} - \frac{v_2}{R_2} = 0$$

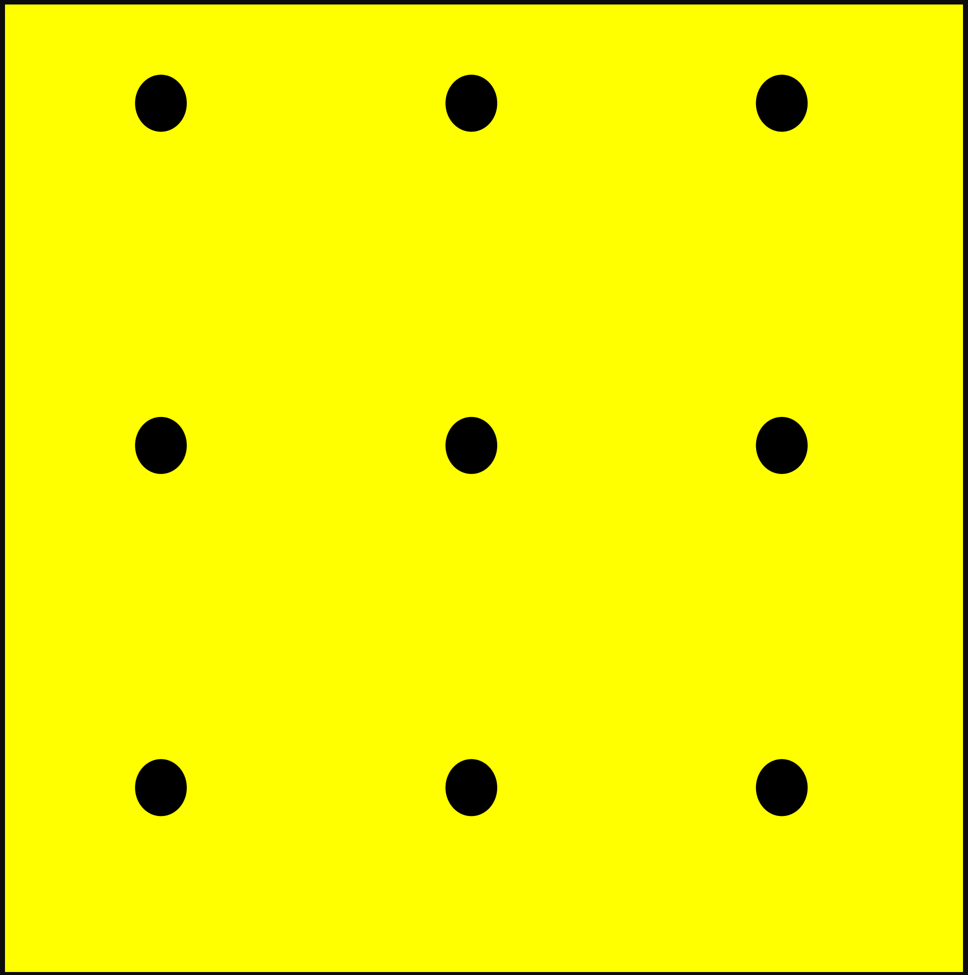
Substituting values, we get the two equations:

$$v_1 + v_2 = 6$$

$$v_1 - \frac{1}{2}v_2 = 0$$

Solving, we get $v_1 = 2$ volts and $v_2 = 4$ volts.

Check for Unnecessary Constraints



Discuss

- Discuss the problem with other people.
- Sometimes just describing the problem gives you new insights.
- When discussing problems be sure to listen carefully for ideas.

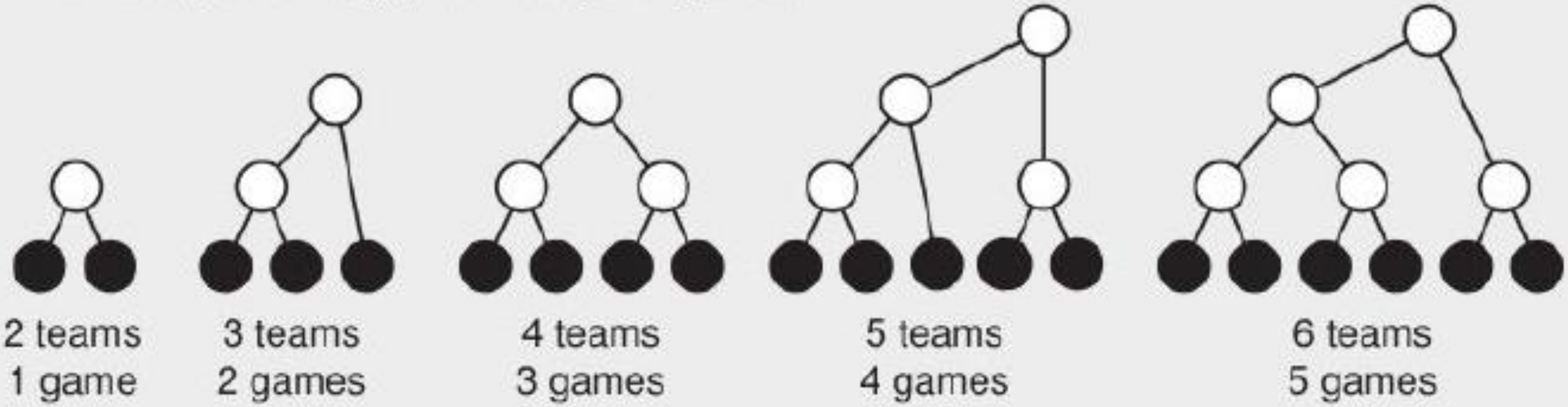
Solve a Scaled-Down Version of the Problem

Example 3.8

Number of Games in a Tournament, Solved Another Way

Try solving the problem of determining the total number of games played in a tournament, presented earlier in Example 3.4, by first solving simple cases with only a few teams entered.

Solution The sketch below solves the problem for the simple cases of 2 through 6 teams, which are small enough to be solved by sketching the tournament brackets in their entirety and simply counting the games.



Try Solving a Similar but Related Problem

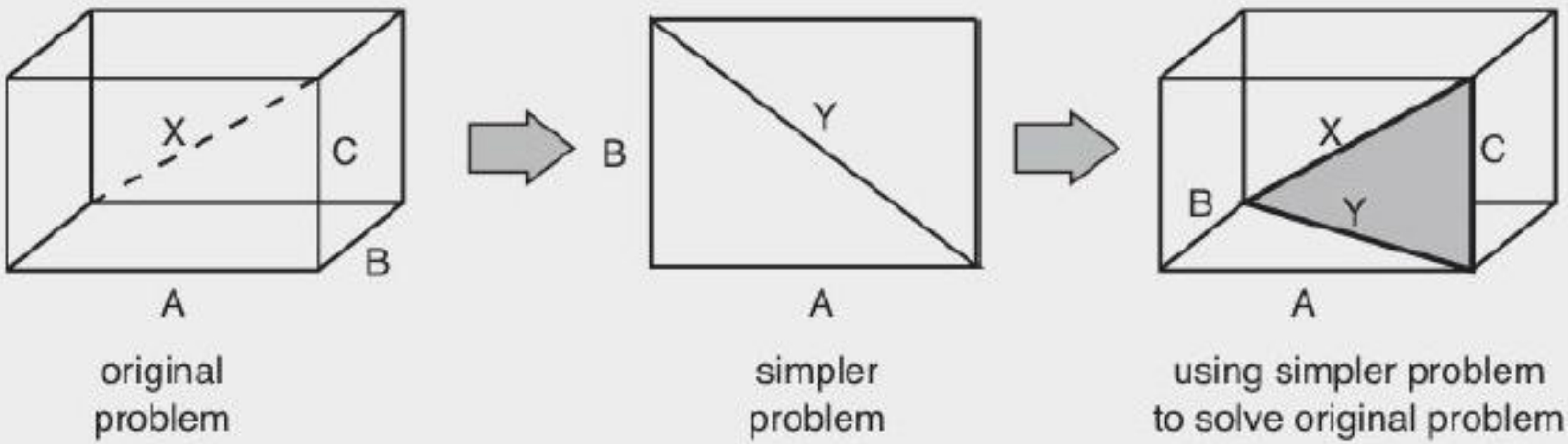
Example 3.9

Finding the Diagonal of a Box

Given a rectangular box with dimensions A, B, and C, what is the length of the body diagonal?

Try Solving a Similar but Related Problem

Solution In *How to Solve It* [Pol45], the author describes an imaginary discussion between a teacher and a student who is stuck on this problem. The teacher asks the student if he can't come up with a simpler problem that reminds him of the original problem. With some prodding, the student comes up with the problem of finding the length of the diagonal of a rectangle, which he knows can be solved using the Pythagorean theorem. Then the teacher asks the student if he can find a way to apply this result to the solution of the original problem. Now the student sees a path to a solution, illustrated below. Do you see it?



Use Models

Example 3.10 **Hobbits and Orcs**

Three Hobbits and three Orcs are on one side of a river. They all have to get across, using a canoe that can hold only two of these creatures at a time. At no point can more Orcs than Hobbits be left on one side of the river, as the Orcs would kill the Hobbits. Develop a plan for transporting all six creatures across the river that keeps the Hobbits alive.

Solution Using simple physical models such as different coins for the Hobbits and Orcs is much easier than trying to solve this problem in your head, and more convenient than using pencil and paper. This problem is also often used to illustrate the point that sometimes you need to take a step that seems to move you further *away* from the goal in order to solve the problem.

Note that the Hobbit/Orc version of this problem is a more "culturally sensitive" version presented in [And80] of a classic artificial intelligence problem originally called "missionaries and cannibals" [NS63].

Guess and Check

Example 3.11

Guess and Check to Find a Root of a Polynomial

Find a value for x, accurate to one decimal place, that solves the following equation:

$$f(x) = x^3 - 7x^2 + 20x - 100 = 0$$

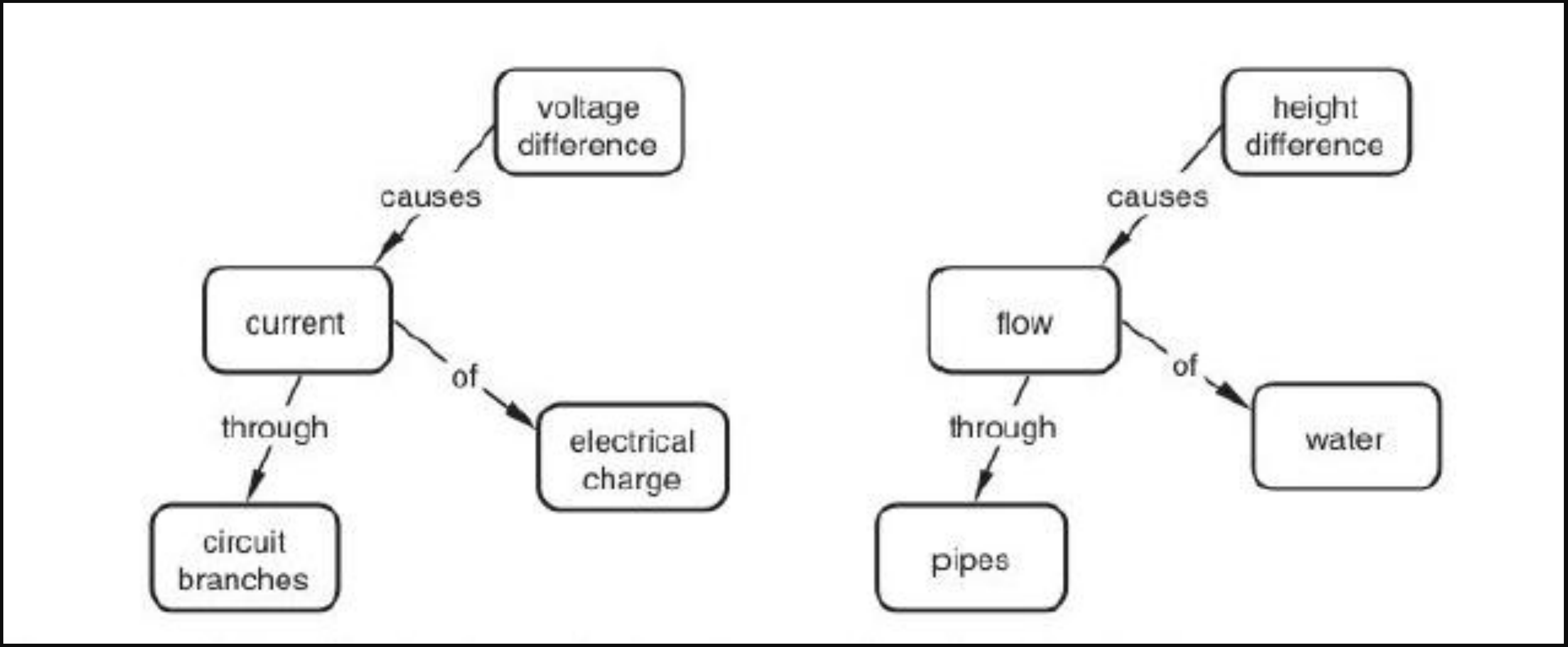
Guess and Check

Guess	x	$f(x) = x^3 - 7x^2 + 20x - 100$
1	0	−100
2	10	400
3	5	−50
4	7.5	78.1
5	6.25	−4.3
6	6.4	3.4
7	6.3	−1.8

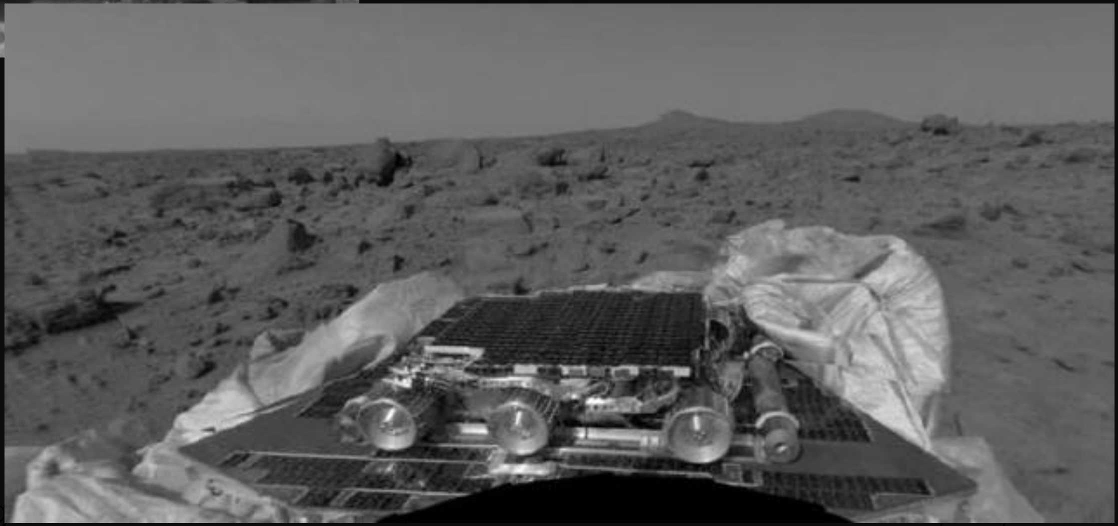
Thus $x = 6.3$ is an accurate solution to the equation to within one decimal place. Note that there are a number of ways that we could have been "smarter" with our guesses. For example, we could have plotted the function at $x = 0$, $x = 5$, and $x = 10$ and then interpolated to see where the plot crosses the x-axis. This would have saved us several guesses.

Use and Analogy

Water flow is often used as an analogy to describe the flow of current in an electrical circuit.



Change Your Perspective



Also,

- Look at the big picture (especially if stuck on a small detail, is it really important?).
- Do the easy parts first (more manageable).
- Plug in the numbers (can concrete up a problem).
- Keep track of progress (a to-do list).
- Change the representation.
- Re-plan (often the plan is not good).
- Pat attention to hunches (gut-feeling).
- Take a break (this works for me).