Chapter 2

Chapter 2

Problem 1: Short-Term Memory

Solution 1.1: Jay's Solution

Solving any problem that involves more than "seven plus or minus two" unrelated concepts will bring the limitations of short-term memory into play. If we can't easily create chunks of information that we can recall for solving the problem, writing the concepts down on paper--as we do when solving arithmetic problem--usually helps. Some examples include:

- sorting a list of items into alphabetical order
- solving an algebra problem for an unknown variable
- getting directions to a destination in an unfamiliar city (but you might be able to remember direction to a location in a familiar city because you can chunk part of the route)

Problem 2: Why Won't My Car Start?

Solution 2.1: Jay's Solution

Here are some possibilities:

```
IF the headlights won't turn on,
THEN the battery might be dead

IF the starter motor seems to be turning slowly,
THEN the battery is probably weak

IF the starter motor turns normally and you don't smell gas,
THEN you might be out of gas

IF the starter motor turns normally and you smell gas,
THEN the engine might be flooded

IF the fuel indicator is on "empty"
THEN you are out of gas

IF you think you're out of gas
THEN add some to the tank

IF you suspect that the battery is dead or weak
THEN try charging it

IF you think that the engine is flooded
THEN wait a few minutes and try again
```

Note that there is a web site for the "eXpertise2Go" expert system that has a demonstration of an online system to diagnose why a car won't start using a set of rule. The demo is at: http://expertise2go.com/webesie/car/ and the set of rules may be viewed at http://www.expertise2go.com/webesie/e2gdoc/e2gref.htm#KBASEEX

Problem 3: Planning a Solution to a Problem

Solution 3.1: Jay's Solution

We can find examples of each of these strategies in the process of designing the first prototype version of our first year engineering course at Notre Dame.

- An example of *divide-and-conquer*
  is the basic modular design of the course, where we had a team of faculty where each member was responsible for developing the details of one of four modules.

- An example of *working forwards*
  was in the design of some of the lab exercises. For example, we were given a room with certain limitations, and sometimes moved forward beginning from those constraints.

- We often adopted a *working backwards*
  strategy for fleshing out the details of the modules. Typically, we would start with the objectives that we wanted the students to meet by the end of the module, then define intermediate milestones, and then complete the schedule of lectures and assignments.

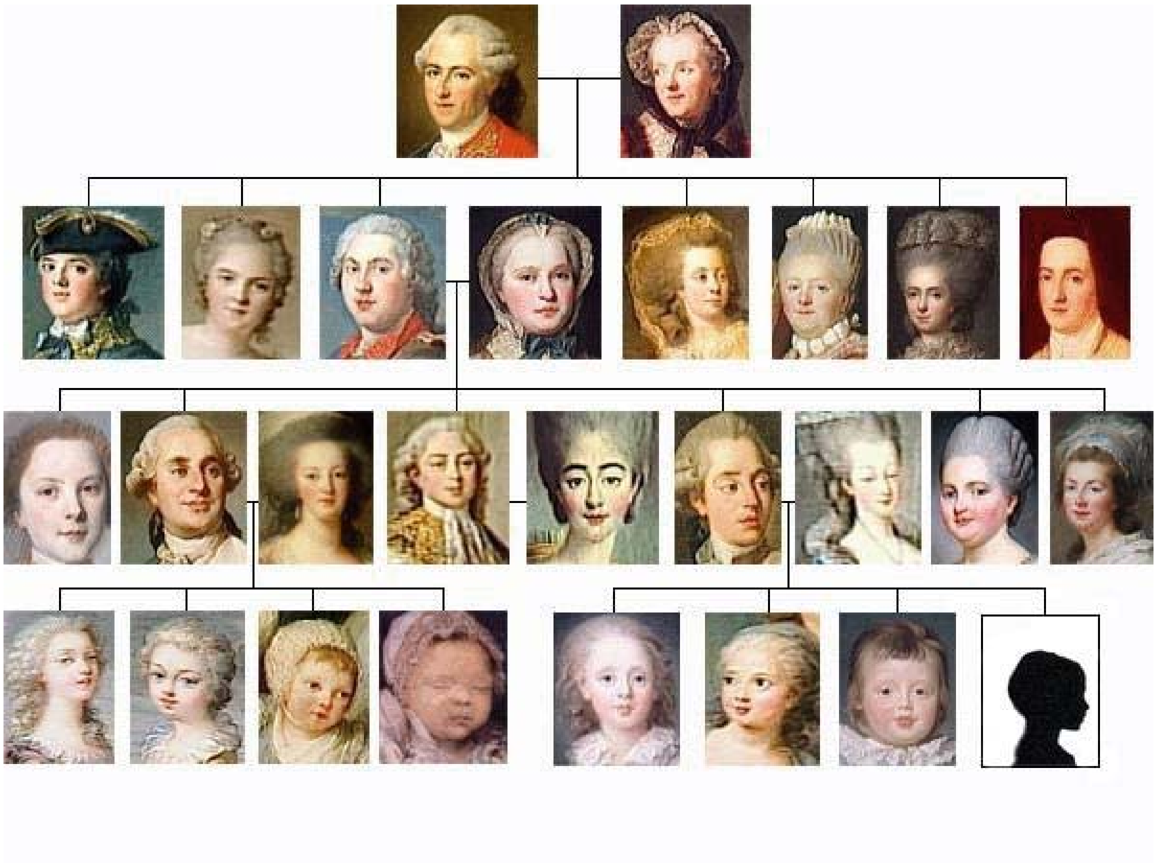
- An example of *working from the middle*
  was in the design of a project module that incorporated lightweight structures or trusses. We knew that we wanted to incorporate material developed for a high school outreach project using K'NEX kits. From this, we worked forward to a set of project goals, and then backwards towards lectures and assignments.

Problem 4: Using Graphs to Convey Information

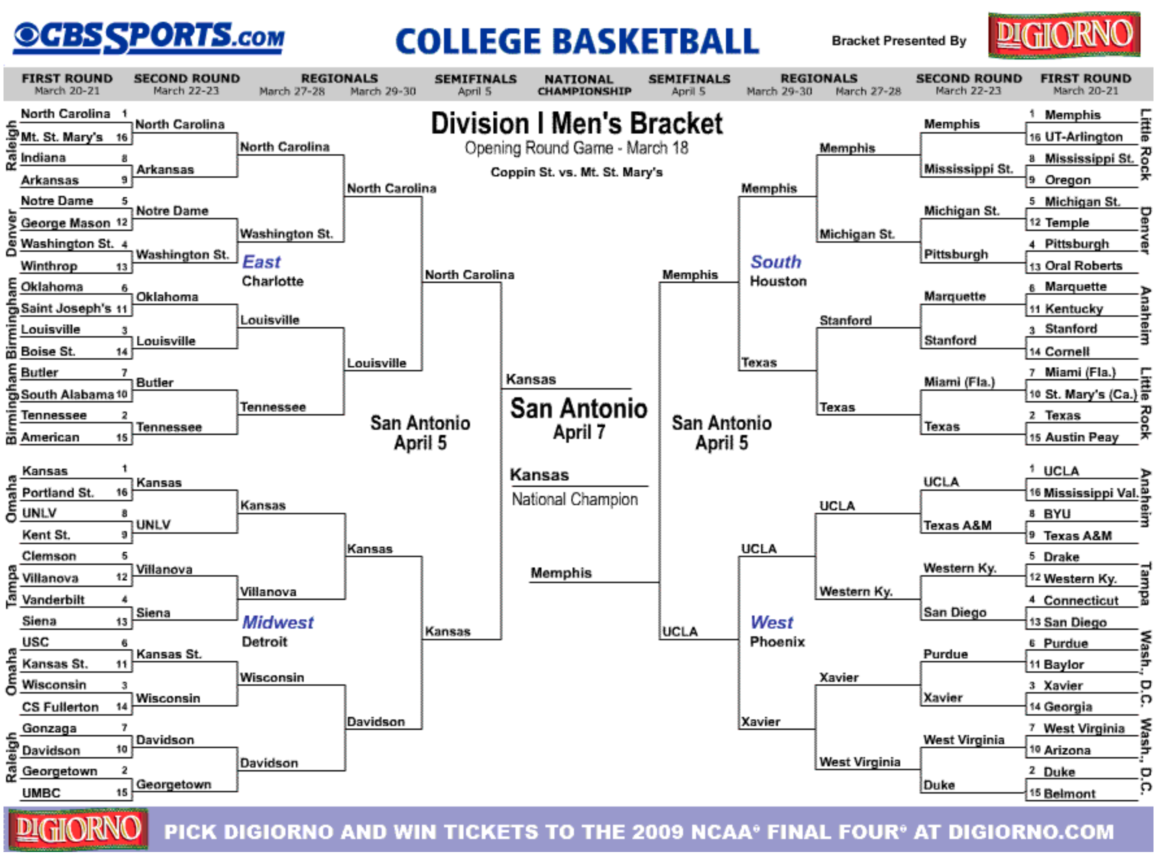
Solution 4.1: Jay's Solution

Some examples include:

- a family tree, such as this one of the family of the lineage of Marie Therese Charlotte of France, Madame Royale, first child and first daughter of King Louis XVI. of France and Marie Antoinette of Austria, Queen of France. (From WikiMedia Commons)



- brackets from a basketball tournament, such as this one of the NCAA men's basketball tournament

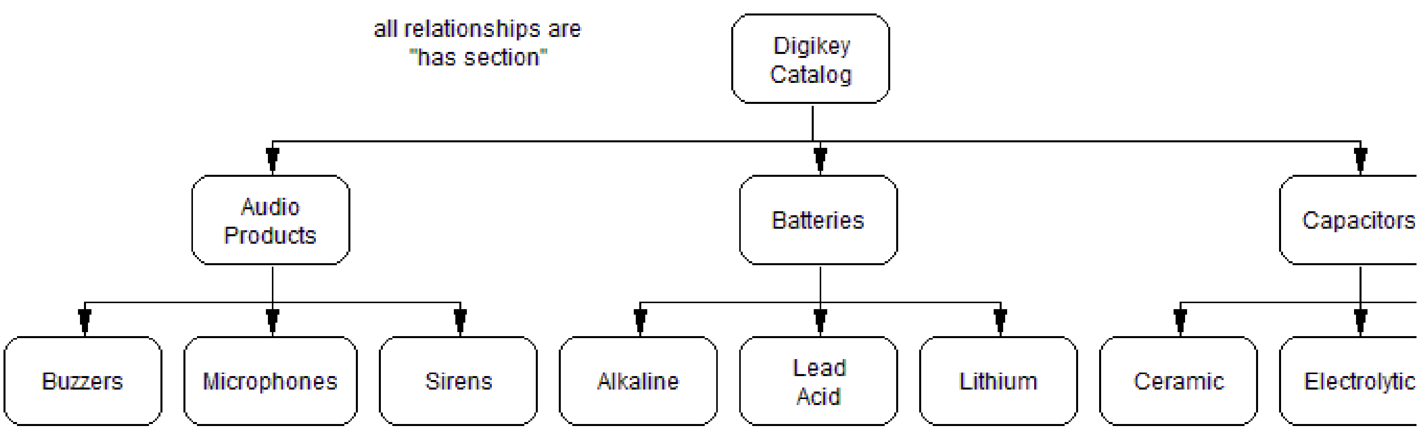


- a distance map between cities (the links don't represent actual roads)

Problem 5: Organization of a Parts Catalog

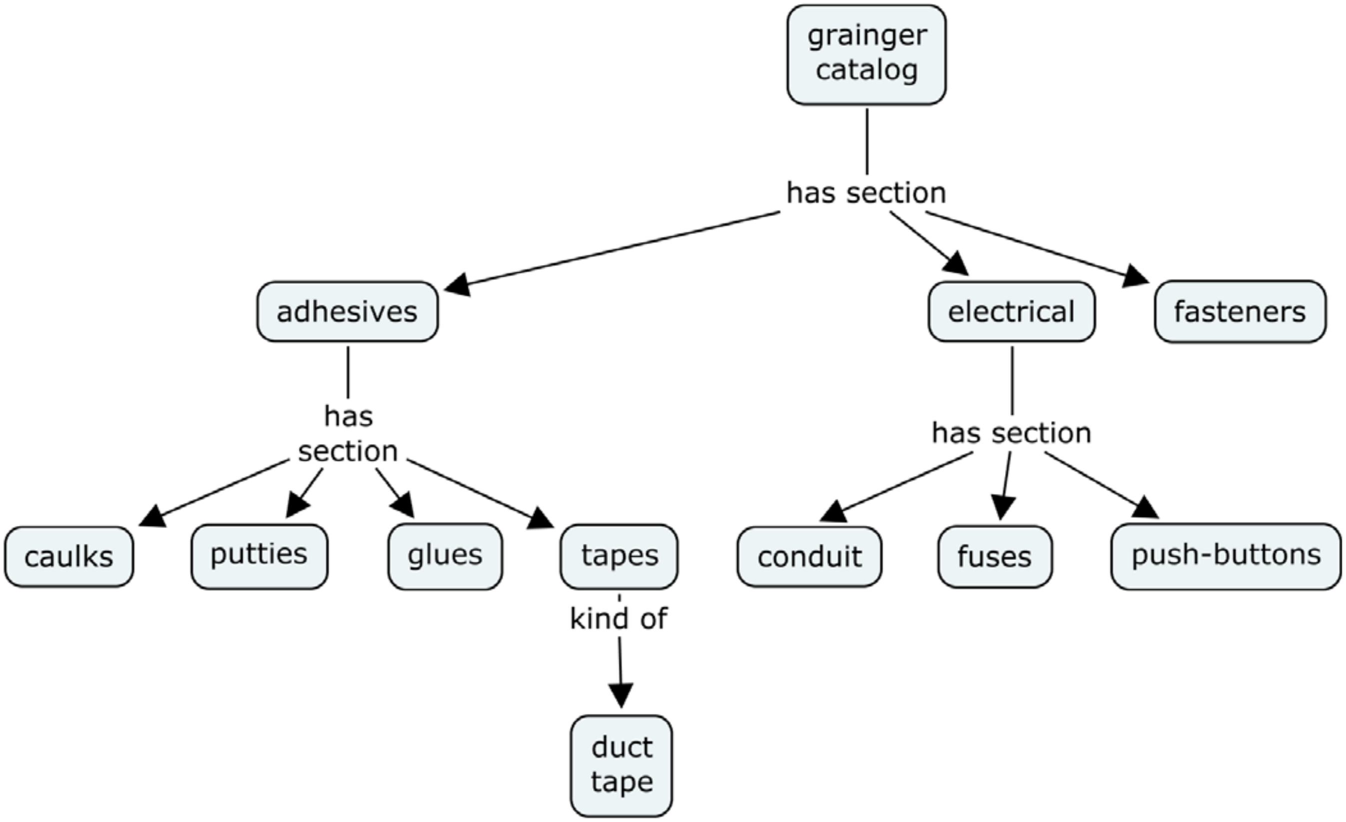
Solution 5.1: Digikey Catalog

Below is a diagram of a small part of the Digikey catalog (http://www.digikey.com)



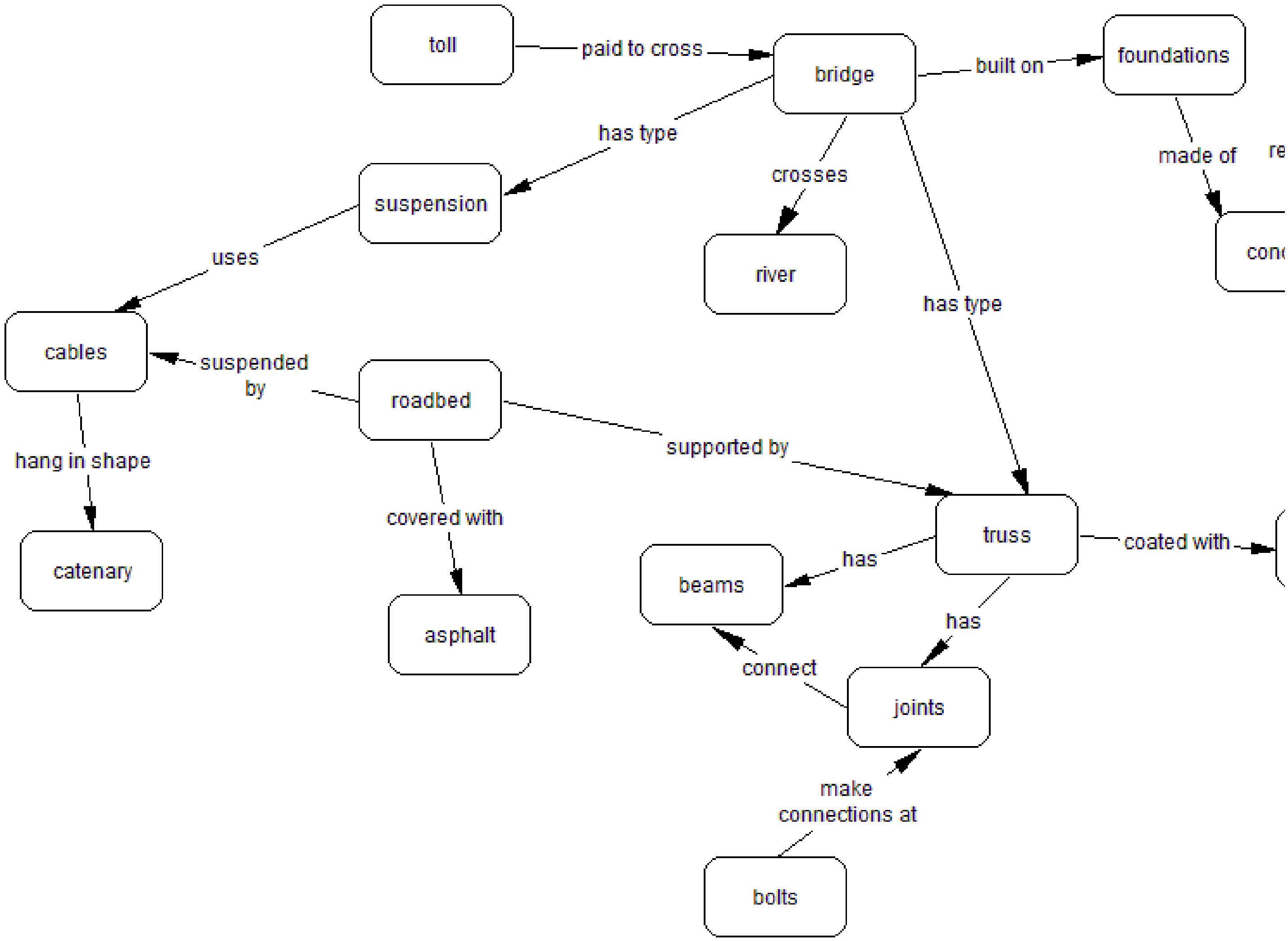
Solution 5.2: Grainger

Here's an example from Grainger Industrial (http://www.grainger.com)



Problem 6: Construct a Concept Map: River, Beam, etc.

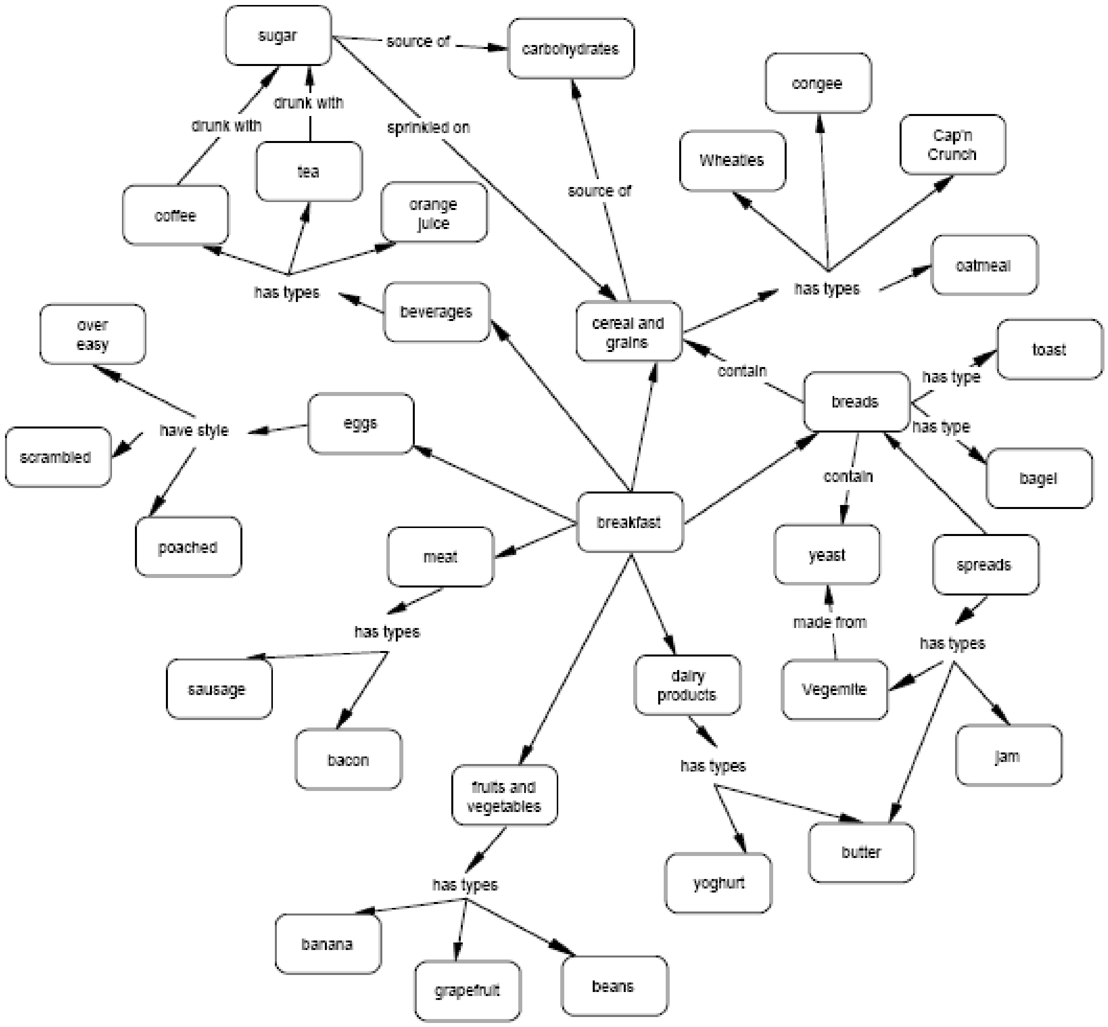
Solution 6.1: Jay's Solution



Problem 7: Construct a Concept Map: Sausage, Vegemite, etc.

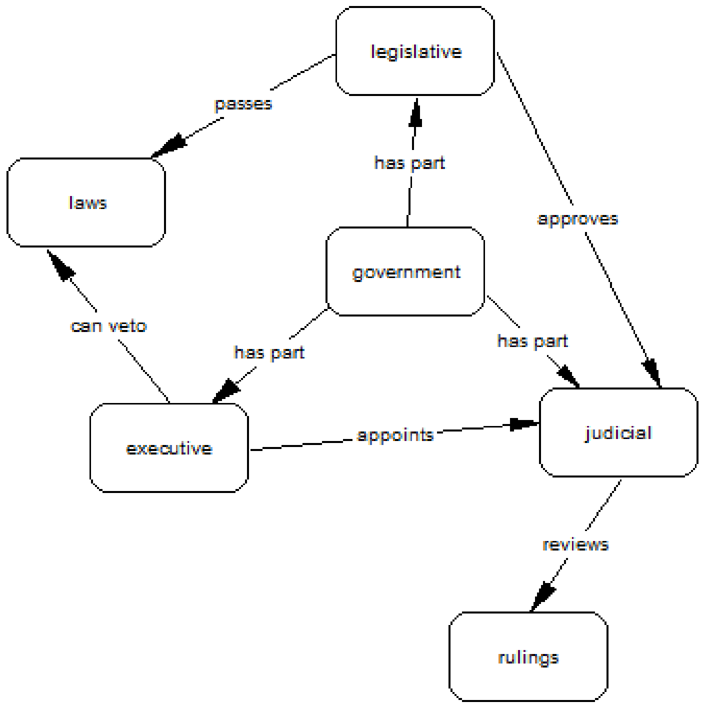
Solution 7.1: Jay's Solution

Here's one possibility:



Problem 8: Concept Map: U.S. Government

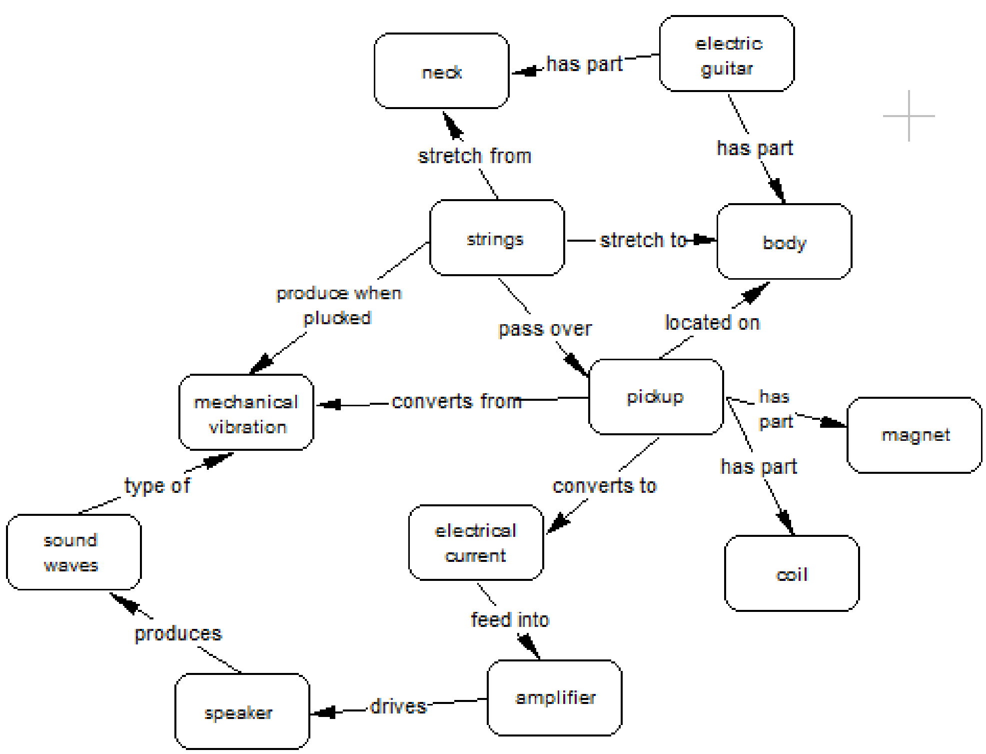
Solution 8.1: Jay's Solution



Problem 9: How Stuff Works

Solution 9.1: Electric Guitar

Here's an example for an electric guitar:



Hierarchy is used in stating that the guitar "has parts" neck and body, and that the pickup "has parts" coil and magnet.

Problem 10: Product Specifications and Design Objectives

Solution 10.1: iPhone

A great example is the specs page for Apple's iPhone 3G (http://www.apple.com/iphone/specs.html)

A few examples of specifications listed as constraints are:

- height = 4.5 inches
- Frequency response: 20Hz to 20,000Hz (20Hz $\leq$ f $\leq$ 20,000Hz)
- Video Playback $\leq$ 7 hours

An unstated objective is that the design should be aesthetically appealing

Technical Specifications

Size and weight[1]

Height: **4.5** inches (115.5 mm)

Width: **2.4** inches (62.1 mm)

Depth: **0.48** inch (12.3 mm)

Weight: **4.7** ounces (133 grams)

Color

- 8GB model: Black
- 16GB model: Black or white

Capacity[2]

- 8GB or 16GB flash drive

Cellular and wireless

- UMTS/HSDPA (850, 1900, 2100 MHz)
- GSM/EDGE (850, 900, 1800, 1900 MHz)
- Wi-Fi (802.11b/g)
- Bluetooth 2.0 + EDR

GPS

- Assisted GPS

In the box

- iPhone 3G
- Stereo Headset with mic
- Dock Connector to USB Cable

Display

- 3.5-inch (diagonal) widescreen Multi-Touch display
- 480-by-320-pixel resolution at 163 ppi
- Support for display of multiple languages and characters simultaneously

Audio

- Frequency response: 20Hz to 20,000Hz
- Audio formats supported: AAC, Protected AAC, MP3, MP3 VBR, Audible (formats 1, 2, and 3), Apple Lossless, AIFF, and WAV
- User-configurable maximum volume limit

Headphones

- Stereo earphones with built-in microphone
- Frequency response: 20Hz to 20,000Hz
- Impedance: 32 ohms

Video

Video formats supported: H.264 video, up to 1.5 Mbps, 640 by 480 pixels, 30 frames per second, Low-Complexity version of the H.264 Baseline Profile with AAC-LC audio up to 160 Kbps, 48kHz, stereo audio in .m4v, .mp4, and .mov file formats; H.264 video, up to 2.5 Mbps, 640 by 480 pixels, 30 frames per second, Baseline Profile up to Level 3.0 with AAC-LC audio up to 160 Kbps, 48kHz, stereo audio in .m4v, .mp4, and .mov file formats; MPEG-4 video, up to 2.5 Mbps, 640 by 480 pixels, 30 frames per second, Simple Profile with AAC-LC audio up to 160 Kbps, 48kHz, stereo audio in .m4v, .mp4, and .mov file formats

Camera and photos

- 2.0 megapixels
- Photo geotagging
- iPhone and third-party

Connectors and input/output

External buttons and controls

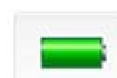
Sensors

- Accelerometer
- Proximity sensor
- Ambient light sensor

Power and battery

- Built-in rechargeable lithium-ion battery[3]
- Charging via USB to computer system or power adapter
- Talk time:[4]
  Up to 5 hours on 3G
  Up to 10 hours on 2G

- Standby time: Up to 300 hours[5]
- Internet use:
  Up to 5 hours on 3G[6]
  Up to 6 hours on Wi-Fi[7]



- Video playback: Up to 7 hours[8]
- Audio playback: Up to 24 hours[9]

Mac system requirements

- Mac computer with USB 2.0 port
- Mac OS X v10.4.10 or later
- iTunes 7.7 or later

Windows system requirements

- PC with USB 2.0 port
- Windows Vista; or Windows XP Home or Professional with Service Pack 2 or later
- iTunes 7.7 or later

Problem 11: Form, Purpose, and Environment

No solutions posted for this problem.

Problem 12: Purchase of a Faulty Product

No solutions posted for this problem.

Problem 13: An Ill-Conceived Product

No solutions posted for this problem.

Problem 14: Design Alternatives and Constraints

No solutions posted for this problem.

Problem 15: Snow Removal System

No solutions posted for this problem.

Problem 16: The X Prize

No solutions posted for this problem.

Problem 17: Alternative Pump Designs

No solutions posted for this problem.

Problem 18: Gear Pumps

Solution 18.1: Solution from Viking Pump, Inc.

A great resource for solving this problem is the "Pump School" web page on "Rotary Pump Principles" from Viking Pump, Inc. (http://www.pumpschool.com/principles/). Below are excerpts from their page:

External Gear Pump:

How External Gear Pumps Work

External gear pumps are similar in pumping action to internal gear pumps in that two gears come into and out of mesh to produce flow. However, the external gear pump uses two identical gears rotating against each other -- one gear is driven by a motor and it in turn drives the other gear. Each gear is supported by a shaft with bearings on both sides of the gear.

< Click Here to Animate >

1. As the gears come out of mesh, they create expanding volume on the inlet side of the pump. Liquid flows into the cavity and is trapped by the gear teeth as they rotate.

2. Liquid travels around the interior of the casing in the pockets between the teeth and the casing -- it does not pass between the gears.

3. Finally, the meshing of the gears forces liquid through the outlet port under pressure.

Because the gears are supported on both sides, external gear pumps are quiet-running and are routinely used for high-pressure applications such as hydraulic applications. With no overhung bearing loads, the rotor shaft can't deflect and cause premature wear.

Advantages

- High speed
- High pressure
- No overhung bearing loads
- Relatively quiet operation
- Design accommodates wide variety of materials

Disadvantages

- Four bushings in liquid area
- No solids allowed
- Fixed End Clearances

Internal Gear Pump (also called a "gerotor"):

How Internal Gear Pumps Work

1. Liquid enters the suction port between the rotor (large exterior gear) and idler (small interior gear) teeth. The arrows indicate the direction of the pump and liquid.

2. Liquid travels through the pump between the teeth of the "gear-within-a-gear" principle. The crescent shape divides the liquid and acts as a seal between the suction and discharge ports.

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3. The pump head is now nearly flooded, just prior to forcing the liquid out of the discharge port. Intermeshing gears of the idler and rotor form locked pockets for the liquid which assures volume control.

4. Rotor and idler teeth mesh completely to form a seal equidistant from the discharge and suction ports. This seal forces the liquid out of the discharge port.

Advantages

- Only two moving parts
- Only one stuffing box
- Non-pulsating discharge
- Excellent for high-viscosity liquids
- Constant and even discharge regardless of pressure conditions
- Operates well in either direction
- Can be made to operate with one direction of flow with either rotation
- Low NPSH required
- Single adjustable end clearance
- Easy to maintain
- Flexible design offers application customization

Disadvantages

- Usually requires moderate speeds
- Medium pressure limitations
- One bearing runs in the product pumped
- Overhung load on shaft bearing

Wikipedia also has a very good entry with illustrations (http://en.wikipedia.org/wiki/Gear_pump)

Chapter 3

Chapter 3

Problem 1: Concept Map: Expertise and the Learning Process

No solutions posted for this problem.

Problem 2: Concept Map: Engineering Disciplines

No solutions posted for this problem.

Problem 3: Studying with Music

Solution 3.1: Shannon's Solution

Yes.

Problem 4: Levels of Understanding: Making Copies

Solution 4.1: Jay's Solution

1. Mike gets a job working as a copyboy. Today is his first day, and within the first hour, the copier runs out of paper. Mike refills the copier.

- There may have been a small amount of *analysis* required to determine that the copier was out of paper, but more likely it was simply *comprehending* and error message on the control panel of the copier, and then *applying* a well-known procedure.

2. Mike needs to make copies of 10 articles for one of his business' employees.

- Suppose that Mike was simply told to make copies of the 10 articles. He would *analyze* this problem and then *synthesize*
 a plan for doing this in the most efficient way. For example, if several articles were next to each other in the same journal or magazine, he could copy them together, while if they were in different journals, he would

proceed differently.

3. The papers jam in the middle of making the copies. Mike needs to fix the copier and finish making the copies. Luckily, the screen on the copier gives explicit directions to remove the paper jam.

- Since the screen gives explicit directions, this most likely just involves *comprehension* of the message and then *application* of a well-known procedure.

4. Mike is having a very trying day. When he tries to remove the paper jam, he accidentally changes some settings on the copier and the copies are not the same as the original ones. Mike rectifies this situation.

- Here, Mike had to *analyze* the situation to determine what the problem was.

5. A message shows up on the copier panel telling Mike that the toner is low. He must fix this problem as well.

- This just involves *comprehending* the message and *applying* the procedure to fix it.

6. Finally, Mike's day ends. He is upset with the issues with the copier, and he is contemplating getting a new job that he enjoys more. In the process of finding a job, he does a lot of research and eventually finds the perfect job: as a go-kart tester.

- A classic case of Mike *evaluating* his options.

Problem 5: Levels of Understanding: Making Up Your Own Questions

No solutions posted for this problem.

Problem 6: Hmm . . .

Solution 6.1: Jay's Solution

This problems test *application* of Bloom's Taxonomy.

Problem 7: Practice with the Problem-Solving Framework

Solution 7.1: Jay's Solution

Part 1:

In one long-distance phone call, Amy talked to her parents for twice as long as her brother talked. Her sister talked for 12 minutes longer than Amy. If the phone call was 62 minutes long, how long did each person talk on the phone?

Given:
A set of 3 relationships referring to how long Amy, her brother and her sister talked on a single phone call to their parents.

Find: How long each person spoke

Plan

1. Define variables for how long each person spoke

2. Express the 3 relationships in the word problem in terms of these 3 variables
3. Solve the 3 equations in 3 variables using substitution

Analysis:

- Variables for minutes on phone: Amy (A), brother (B), sister (S)

- Express relationships and solve

Sub (1) into (2)

$$(4) \quad S = 12 + 2B$$

Sub (1) & (4) into (3)

$$2B + B + (12 + 2B) = 62$$
$$5B + 12 = 62$$
$$5B = 50$$
$$\boxed{B = 10}$$
$$(1) \rightarrow \boxed{A = 2B = 20}$$
$$(2) \rightarrow \boxed{S = 12 + A = 32}$$

Thus Amy was on the phone for 20 min, her brother for 10, and her sister for 32.

Comments:

* The main heuristic employed was **restating in simpler terms**, rewriting the word problem as an equation.

Part 2:

Roberto needs to draw a line that is 5 inches long, but he does not have a ruler. he does have some sheets of notebook paper that are each 1.5 inches wide and 11 inches long. Describe how Roberto can use the notebook paper to measure 6 inches.

Given: strips of paper 1.5 x 11.0 inches

Find: a process to measure lengths of either 5 or 6 inches

Plan:

- use the fact that we can make measurements in increments of 1.5 inches from the width of the paper strips
- use the fact that we can subtract known lengths from 11 to get other lengths
- note that 5 + 6 = 11

Analysis:

1. draw a line 6 inches long using 4 widths of paper (1.5 X 4 = 6)

2. subtract a measurement 6 inches long from the length of paper to get 5 inches (11 - 6 = 5)

Comments:

Solving this problem used a combination of **working forward** by starting with a strip of paper and seeing that we could use it to measure out 6 inches, and then **working backwards** to get the 5 inches from the 6 inch measurement and the fact that the paper is 11 inches long. There was also implicit **solving a simpler problem** from the hint that was given asking how to find a line 6 inches long. I think the problem would have been considerably tougher if this hint weren't given!

Part 3:

As the hands of a clock move from 6:00 AM to 6:00 PM, how many times to the hands form a right angle?

Given: A clock with hour and minute hands

Find: How many times the hands form a right angle between 6 AM and 6 PM

Plan:

- Try a few test cases
- Look for a pattern
- Check for exceptions

Analysis:

In general, the hands will form a right angle whenever the minute hand is pointing at a number approximately 3 units ahead (clockwise) or 9 units ahead of the hour hand. In most cases, the minute hand won't be pointing exactly at a digit because the hour hand moves. For example, between 12:00 and 1:00, the two times that we get a 90 degree angle between the hands is approximately 12:16 and 12:49


Thus in *most*
cases, there will be two times per hour when the hands form a right angle. There's one exception, however. Between 8:00 and 9:00, the first time that the clocks form a right angle is approximately 8:26, but the second time *is 9:00*, which is of course also the first time that the hands form a right angle between 9:00 and 10:00. Thus there are only 3 times that they form a right angle between 8:00 and 10:00, rather than the usual 4.

The same thing happens at 3:00. Thus over the 12 hour period, there are 12*2 - 2 = 22 times that the hands form a right angle.

Comments:

The heuristics that I used in solving this problem included:

- Draw a picture/Use models--I needed to look at an actual clock to solve this; couldn't do it in my head!
- Divide and conquer--I broke the problem down into two main cases: what happens within a typical hour, and are there any exceptions.
- Solved a related problem?--I noticed the exception at 9:00 first, and from this determined that there would also be an exception at 3:00.

Part 4:

Alice, Nathan, and Marie play in the school band. One plays the drum, one plays the saxophone, and one plays the flute. Alice is a senior. Alice and the saxophone player practice together after school. Nathan and the flute player are sophomores. Who plays which instrument?

Given: a set of clues regarding Alice, Nathan, and Marie, and the instruments drum, sax, and flute.

*Find: who plays which instrument

Plan:

- Draw a table with people labeling each column and instruments labeling each row.
- Using the table to keep track, use the clues to establish who is or is not playing which instrument. Sort of like playing Sudoku!

Analysis:

We'll start with a blank table, where a given cell notes whether or not we know if the person in the column heading plays the instrument in the row heading. A blank means we don't know, a T means "true" and an F means "false"

	Alice	Nathan	Marie
Drum			
Sax			
Flute			

From the clue, "Alice and the sax player practice together," we know that Alice doesn't play sax:

	Alice	Nathan	Marie
Drum			
Sax	F		

Flute			

From the clues, and "Nathan and the flute player are sophomores," we know that Nathan doesn't play flute, and from "Alice is a senior" Alice doesn't play flute, either, so she must play drum.

	Alice	Nathan	Marie
Drum	T		
Sax	F		
Flute	F	F	

Assuming that a person only plays one instrument in the band, we now know enough to fill in the rest of the table:

	Alice	Nathan	Marie
Drum	T	F	F
Sax	F	T	F
Flute	F	F	T

Thus Alice plays drum, Nathan plays sax, and Marie plays flute.

Comments:

Heuristics used in solving this problem were:

- Draw a picture--using the table
- Divide and conquer--solve for Alice first and then use this to help conquer the rest of the problem
- Working forward and backward--in the application of clues

Problem 8: Assumptions and Approximations in Solving Energy Problems

Solution 8.1: Jay's Solution to Part 3 (Reese's Cups)

Part 1:

Part 2:

Part 3:

Given: My favorite candy (Reese's Cups) and exercise through walking

Find: How far you would have to walk to burn the calories contained in the candy

Plan:

- Look up how many calories in a serving of Reese's Cup
- Look up how many calories burned by walking a given distance

Analysis:

According to Hershey's web site (http://www.hersheys.com/products/details/reesespeanutbuttercups.asp), a single serving of Reese's Cups has 260 calories.

Using the chart for calories burned per mile to About.com (http://walking.about.com/cs/howtoloseweight/a/howcalburn.htm), I'll estimate approximately 85 calories per mile. Thus to burn 260 calories would require walking 260/85 = approximately 3 miles.

Comment:

Given that the closest store that sells Resse's Cups is approximately 3 miles from my house, this works out pretty well!

Problem 9: Storing Text on a DVD

Solution 9.1: Jay's Solution

Given: that a letter requires 1 byte of data and DVDs for storage

Find: how many DVDs it would take to store all of the text in books in your school's library

Plan:

- Look up how many bytes per DVD
- Estimate how many bytes of text in all the books in your library
  - How many books in the library?
  - How many pages per book?
  - How many letters per page?
    - How many letters per word?
    - How many words per line?
    - How many lines per page?
- Solve

Analysis:

According to the DVD Forum FAQ (http://www.dvdforum.org/faq-dvdprimer.htm located via a Google Search on "DVD capacity"), the most common DVD format (single-layer, single-sided) has a storage capacity of 4.7 GB. There are other formats available, but we'll assume this one for our calculations.

According to its web site, the University of Notre Dame Library has approximately 2 million books, not counting microfiche and other media.

After looking at a couple of textbooks and novels, I came up with the following estimates of the number of letters per book (including numbers, spaces, and punctuation).

$$500\,\frac{\text{page}}{\text{book}} \times 30\,\frac{\text{line}}{\text{page}} \times 12\,\frac{\text{word}}{\text{line}} \times 500\,\frac{\text{letter}}{\text{word}} = 900,000\,\frac{\text{letter}}{\text{book}}$$

Rounding this up to 1 million, it would thus take approximately 1 MB to store the text of a typical library book in ASCII format.

A 4.7 GB DVD could thus store approximately 4700 books in ASCII format

Storing 2 million books would thus require 2,000,000/4,700 = **425 DVDs**

Comments:

1 MB per book seems like a reasonable number for ASCII text only. For reference, a PDF file of the full *Introduction to Engineering* textbook including all figures, formatting, fonts, etc, is approximately 10 MB.

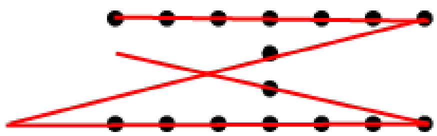
Problem 10: Problem Solving Strategies and Heuristics

Solution 10.1: Jay's Solution

1. Each of the 10 players participates in 9 handshakes, so there are 90 handshake "experiences". But since there are 2 "handshake experiences" for each handshake, there are half as many handshakes as "handshake experiences". Thus there are 45 handshakes.

The heuristic that I used was to first **solve a simpler problem**, namely, thinking about how many handshakes each player experienced. Then I extended this solution to solving the complete problem.

2. Here's a solution:



The heuristic I employed here was **checking for unnecessary constraints**, as one might be tempted to think that there is a constraint to stay within the bounding box of the dots.

Problem 11: Proofs Come in Handy: The Mutilated Chessboard

Solution 11.1: Jay's Solution

1. First, suppose that you have a collection of dominoes that are each exactly the same size as two squares on a chessboard. Is it possible to cover all of the squares on a standard, non-mutilated board with exactly 32 dominoes? How do you know?

- Sure, it's possible. Simply arrange 4 dominoes over each of the 8 rows. The way that we know--and the easiest way to convince someone that you have a solution--is through a demonstration.

2. Now, is it possible to cover all of the squares in a mutilated chessboard with exactly 31 dominoes? How do you know? If it turns out that it's not possible, how can you prove this? Try to solve this problem without looking up the answer, even though it's readily available online.

- It turns out that it's not possible. Here's a proof:
  - Every domino placed on the board will cover 2 squares, one black and one white, regardless of whether it it placed vertically or horizontally.
  - In the mutilated chessboard, when you remove two opposite corners, you are removing two squares of the same color. Thus the number of black squares and white squares are unequal (30 of one and 32 of the other).
  - It's impossible to place 31 dominoes in a manner that would cover unequal numbers of black and white squares, so it is not possible to cover the mutilated chessboard with dominoes.

3. Explain how this problems illustrates both strengths and limitations of of physical demonstrations in solving problems.

- The strategy that we used to determine that there was a solution for the non-mutilated chessboard--providing a physical demonstration--would utterly fail in the case of the mutilated chessboard since there is no solution. Further, we can't really use the fact that we can't easily find a physical demonstration of covering the mutilated board to *prove* that there is no solution, because we would have to try *all possible combinations* for this to suffice as a proof. The logical approach that we used is clearly the better way of demonstrating that no solution exists.

Problem 12: Guess and Check to Determine an Average

Solution 12.1: Converging to an average

We want to find the average of the numbers by using the guess and check method described in the book, and without using division. This method will allow us to converge to a solution.

The first step is to make an educated guess of the average, based on the numbers given. Here, the numbers are: 16, 28, 12, 17, 23, 21. So 20 seems to be a good easy guess to start with. If all six numbers were equal to 20, the numbers' sum would be 120 (20x6). The sum of our original six numbers however is 117. So our guess of 20 is high, and we need to go lower (note that we cannot take the difference, 3, and divide that by 6 to get our average since division is not allowed). So our next guess, based on the feedback from above, can be 19. 19x6 is 114, so we're low, and we try 19.5, which happens to be the correct average. For other examples, you would continue until you reached a point where the error is below your tolerable error level.

This method is of course similar to the binary search.

Problem 13: Rabbit and Turtles

Solution 13.1: Time perspective

Observation:

The key observation for this problem is that rather than track all of the small distances the rabbit travels, we can solve the problem by figuring out how long it takes for the turtles to meet. Once we know this time, we can easily calculate the distance the rabbit travels. This is because the amount of time it takes the turtles to meet each other is the same amount of time that the rabbit travels.

Calculations:

Time for turtles to meet each other:

$300m$ = time travelled $\times$ speed of turtle A + time travelled $\times$ speed of turtle B

$300m$ = time travelled $\times$ 80m/h + time travelled $\times$ 70m/h

$300m$ = time travelled $\times$ 150m/h

$300m/150m/h$ = time travelled

time travelled = $2h$

Distance rabbit travels:

distance = $137m/h$ * time travelled

distance = $137m/h$ * $2h$

distance = $274m$

Problem 14: Does Multiplication Equal Addition?

Solution 14.1: Infinite is not enough!

This problem illustrates than even if you have an infinite number of examples to prove a hypothesis, it still does not make it necessarily true.

In our example, we want to have the following equality, given two numbers a and b: a x b = a + b

Therefore, given a number a, we solve for b: b = a / (a - 1)

That's how the pairs (2, 2), (3, 1.5), etc. are all pairs where their sum is equal to their product. There are therefore an infinite number of such pairs. Concluding that adding and multiplying are equivalent would obviously be a classic example of sophistry.

Another example, actually much simpler: the claim that "all integers are odd" will have an infinite number of examples to try to prove it, but is nonetheless a false statement.

We hope that this problem will help students recognize that a tight view of a system, without seeing the bigger picture, and without accepting counter-examples nor taking in other views, will lead to disastrous results.

Problem 15: What Do These Problems Have in Common?

Solution 15.1: Jay's Solution

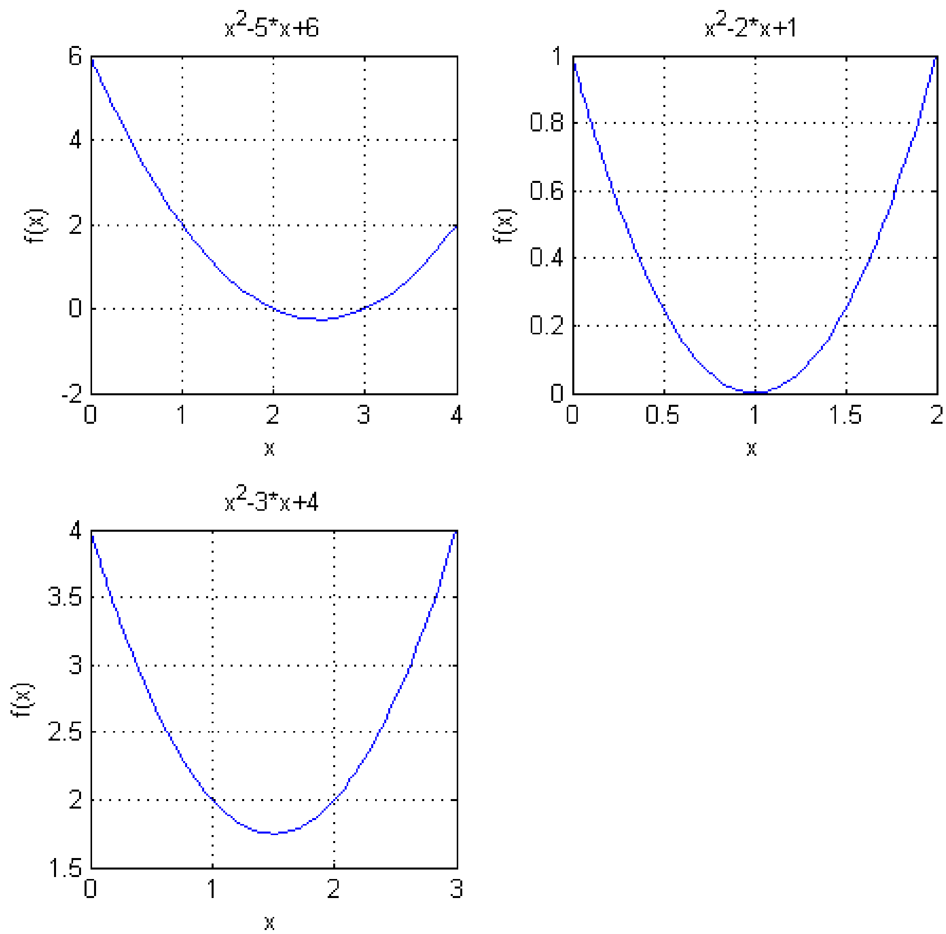
Each of these problems require an *application* level of understanding of basic physical laws.

Problem 16: Graphical Insights on Quadratic Equations

Solution 16.1: Jay's Solution

Using either factoring or the quadratic formula, we find that each of the given polynomials have the real roots listed in the table below, as well as the number of crossings of the x-axis as shown in plots of the equations.

polynomial	real roots	x-axis crossings
$x^2 - 5x + 6$	2,3	2
$x^2 - 2x + 1$	1 (repeated)	1
$x^2 - 3x + 4$	no real roots	0



The MATLAB script used to generate the plots is given below:

```
subplot(2,2,1)
fplot('x^2-5*x+6',[0,4])
xlabel('x')
ylabel('f(x)')
title('x^2-5*x+6')
grid
subplot(2,2,2)
fplot('x^2-2*x+1',[2,0])
xlabel('x')
ylabel('f(x)')
title('x^2-2*x+1')
grid
subplot(2,2,3)
fplot('x^2-3*x+4',[0,3])
xlabel('x')
ylabel('f(x)')
title('x^2-3*x+4')
grid
```

Problem 17: Substitution of Variables to Change a Problem into Something Familiar

Solution 17.1: Change of variable

Here we have an equation where we need to solve for x, but in a form that at first seems complicated. However, upon closer look, we notice that if we use the following variable:

y = x sqrt(x) (1)

our equation then becomes

y^2 - 3y + 2 = 0 (2)

We've effectively reduced the problem to a simple quadratic equation.

Solving equation (2), we get: y = 1 or y = 2

From (1), we get: x = y^(2/3), or x is the cubic root of y.

Therefore: x = 1 or x = 4^(1/3)

Problem 18: Combining Common Sense with Algebra

Solution 18.1: Pragmatic solution

Here we see that we have more unknowns that possible equations. Therefore, some common sense will have to be applied.

Let: r be the number of red beads; w the number of white beads; and b the number of blue beads. From the wording of the problem, we can make a safe assumption that none of those numbers is 0.

Since we have 100 beads, we can write:

> r + w + b = 100 (1)

Given that the red, white, and blue beads weigh 20, 6, and 1 gram, resp., and that their total weight is 200 grams, we can write:

> 20r + 6w + b = 200 (2)

So we have two equations, but three unknowns, which as we know gives one degree of freedom. But let's see if we can analyze this problem a little further.

If we subtract equation (1) from equation (2), we get:

> 19r + 5w = 100 (3)

Solving for w:

> w = 20 - 19r/5 (4)

Here is where we make the following two observation: one is that r, w, and b must of course be integers; and the second is that they must be positive. Therefore, looking at equation (4), we must conclude that r must be divisible by 5. However, it cannot be 10, since that would make w negative. Therefore, r = 5. Plugging into (4), we get w = 1. Plugging r and w into (1) then leads to b = 94.

Therefore, we have 5 red beads, 1 white bead, and 94 blue beads.

Problem 19: A Plan for Applying to College

No solutions posted for this problem.

Problem 20: Gantt Chart

No solutions posted for this problem.

Chapter 4

Chapter 4

Problem 1: WWW Scavenger Hunt

No solutions posted for this problem.

Problem 2: Sagredo, Salviati, and Simplicio Discuss Hybrid Cars

No solutions posted for this problem.

Problem 3: Galileo's Interrupted Pendulum

Solution 3.1: Jay's Solution

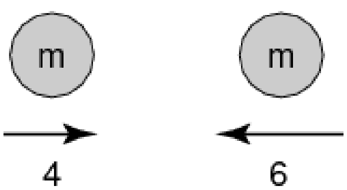
This is a problem to test physical intuition. If the bob were released from a position higher than the pin, then depending upon the initial height, the bob might swing around the pin, causing it to fall or to even wrap around it.

Problem 4: Alternative Solution to Collision Problem

Solution 4.1: Jay's solution

Given: A collision of 2 bodies of equal mass, with initial velocities

$$\begin{aligned} v_{1i} &= 4 \text{ m/s} \\ v_{2i} &= -6 \text{ m/s} \end{aligned}$$



Find: Final velocities after collision, v_{1f} and v_{2f}, using

- conservation of momentum
- conservation of energy

Plan:

1. state the two conservation laws
2. substitute in the initial velocities
3. solve the two equations for the two unknown final velocities

Analysis: State the equations and substitute initial velocities:

$$
\begin{aligned}
\text{conservation of momentum:} \quad & v_{1i} + v_{2i} &= v_{1f} + v_{2f} \\
& 4 - 6 &= v_{1f} + v_{2f} \\
\text{conservation of energy:} \quad & v_{1i}^2 + v_{2i}^2 &= v_{1f}^2 + v_{2f}^2 \\
& 16 + 36 &= v_{1f}^2 + v_{2f}^2
\end{aligned}
$$

Now solve them:

$$
\begin{aligned}
& v_{2f} = -v_{1f} - 2 \\
& v_{1f}^2 + (-v_{1f} - 2)^2 = 52 \\
& 2v_{1f}^2 + 4v_{1f} + 4 = 52 \\
& v_{1f}^2 + 2v_{1f} - 24 = 0 \\
& (v_{1f} + 6)(v_{1f} - 4) = 0 \\
& v_{1f} = -6 \text{ or } 4
\end{aligned}
$$

Of these two mathematically possible solutions $v_{1f} = -6$ makes physical sense. Thus:

$$
\begin{aligned}
v_{1f} &= -6 \text{ m/s} \\
v_{2f} &= 4 \text{ m/s}
\end{aligned}
$$

Comments:
The velocities of the two masses have switched both magnitude and direction after the collision, which is the same result that we got from Example 4.3

Solution 4.2: Second alternative solution

I just needed a case of multiple solutions to test display in assignments.

Problem 5: Perfectly Inelastic Collisions

Solution 5.1: Jay's solution

1.
After the collision, we will have a single combined object of known mass with unknown velocity. Thus we only have one unknown--the velocity--and hence will only need one equation to solve for it. By contrast, in the case of an elastic collision we have two unknowns and hence need two equations.

2. Given:

$$
\begin{aligned}
m_1 = 6 \qquad & v_1 = 3 \\
m_2 = 4 \qquad & v_2 = -5 \\
m_f = 10 \qquad & v_f = -0.2
\end{aligned}
$$

a) Conservation of Momentum:

$$
\begin{aligned}
m_1 v_1 + m_2 v_2 &= m_f v_f \\
6(3) + 4(-5) &= 10(-0.2) \\
-2 &= -2
\end{aligned}
$$

Conservation of momentum holds

b) Conservation of Kinetic Energy:

$$
\begin{aligned}
m_1 v_1^2 + m_2 v_2^2 &= m_f v_f^2 \\
6(3)^2 + 4(-5)^2 &= 10(-0.2)^2 \\
154 &= 0.4
\end{aligned}
$$

Conservation of kinetic energy doesn't hold. Where did the rest of the energy go? After the collision, some of the energy was dissipated as heat.

Problem 6: An Elastic Collision

Solution 6.1: Jay's Handwritten Solution

Given: $m_1 = 1500\,kg$, $V_{1i} = 40\,km/h$
$m_2 = 1800\,kg$, $V_{2i} = 0$

Find: V_{1f}, V_{2f} by:
(1) relative velocity & conservation of momentum
(2) conservation of momentum & conservation of energy

Plan: 1. write equations for 3 laws
2. solve pairs of equations
3. Show that results are (hopefully) the same!

Analysis:

relative velocity: $V_{1i} - V_{2i} = V_{2f} - V_{1f}$
$40 - 0 = V_{2f} - V_{1f}$
$V_{2f} - V_{1f} = 40$

Conservation of momentum: $m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}$
$1500(40) + 1800(0) = 1500 V_{1f} + 1800 V_{2f}$
$1500 V_{1f} + 1800 V_{2f} = 60000$

Conservation of energy: $\frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$
$1500(40)^2 + 1800(0)^2 = 1500 V_{1f}^2 + 1800 V_{2f}^2$

(1) Solve using conservation of momentum and relative velocity

$$1500 V_{1f} + 1800 V_{2f} = 60000 \quad \text{(momentum)}$$

$$V_{2f} - V_{1f} = 40 \quad \text{(rel. velocity)}$$

$$1500 V_{1f} + 1800 (V_{1f} + 40) = 60000$$

$$\boxed{\begin{aligned} V_{1f} &= -3.64 \text{ km/h} \\ V_{2f} &= 36.36 \text{ km/h} \end{aligned}}$$

(2) Solve using conservation of momentum and conservation of energy

$$1500 V_{1f} + 1800 V_{2f} = 60\,000 \quad \text{(momentum)}$$

$$1500 V_{1f}^2 + 1800 V_{2f}^2 = 2400000 \quad \text{(energy)}$$

$$V_{2f} = (-5 V_{1f} + 200)/6$$

$$1500 V_{1f}^2 + 1800 \left[\frac{-5V_{1f} + 200}{6}\right]^2 - 240\,000 = 0$$

$$2750 V_{1f}^2 - 100000 V_{1f} - 40000 = 0$$

From quadratic formula:

$$\boxed{V_{1f} = -3.64 \text{ km/h}} \quad \text{SAME AS ABOVE}$$

Solution 6.2: Solving Equations Symbolically with MATLAB

Script using the Symbolic Math Toolbox in MATLAB

```
m1 = 1500;
v1i = 40;
m2 = 1800;
v2i = 0;

syms v1f v2f;

rv = v1i-v2i+v1f-v2f;
mom = m1*v1i+m2*v2i-m1*v1f-m2*v2f;
eng = m1*v1i^2 + m2*v2i^2 - m1*v1f^2 - m2*v2f^2;

[v1f, v2f] = solve(rv, mom);
v1f_1 = double(v1f)
v2f_1 = double(v2f)

[v1f, v2f] = solve(mom, eng);
v1f_2 = double(v1f)
```

```
v2f_2 = double(v2f)
```

Problem 7: Playing Darts in an Elevator

Solution 7.1: Jay's solution

If the elevator is moving at constant velocity, meaning that you can't feel it accelerating, then you should aim at the target as you normally would on the ground. The dart will have an additional upward velocity component relative to the ground that is due to the motion of the elevator, but you and the dartboard will also have that same additional velocity component.

Problem 8: Billiard Cue Balls

No solutions posted for this problem.

Problem 9: Tricks in Urban Rail Station Design

Solution 9.1: Jay's Solution

Given: A train with initial velocity v_i changes in elevation by $\Delta h = 2\,\mathrm{m}$

Find: Final velocity v_f of train

Plan: Solve the problem using conservation of energy

1. Write equations showing that the decrease in kinetic energy is equal to the increase in gravitational potential energy
2. Solve for v_f in terms of v_i
3. Plot the results

Analysis:

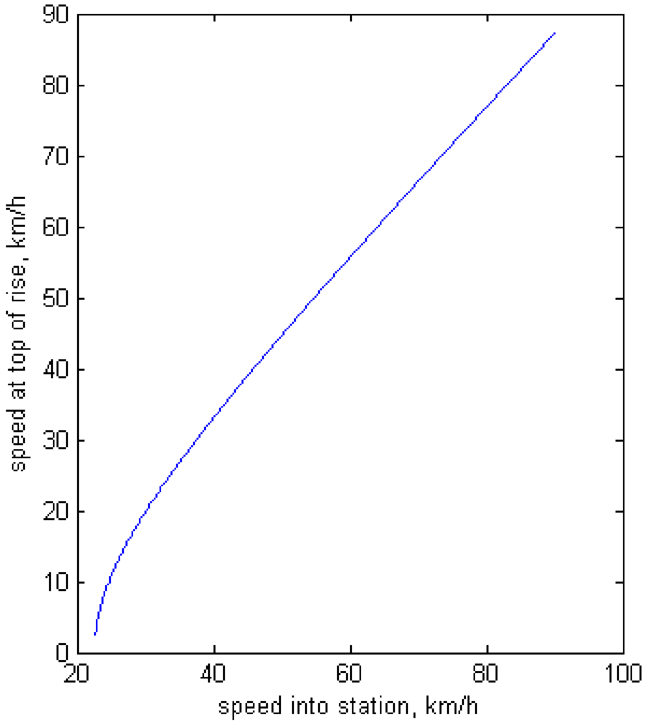
From conservation of energy

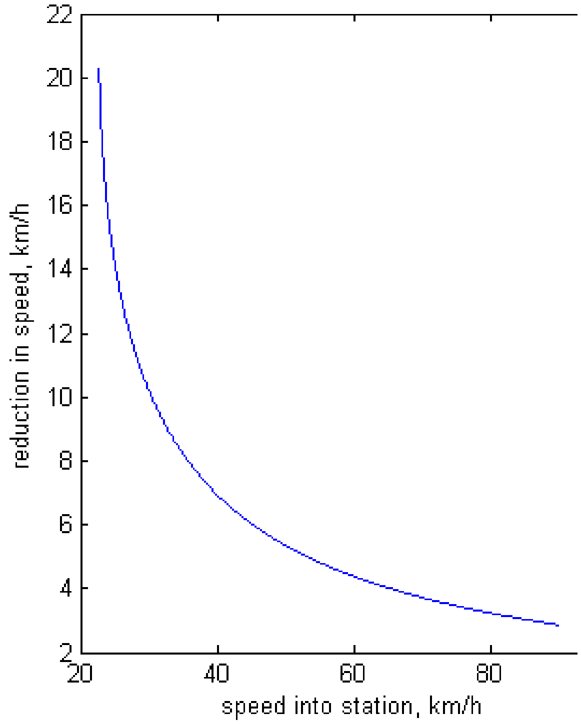
$$\begin{aligned} mg\Delta h &= \frac{1}{2}m(v_i^2 - v_f^2) \\ v_i^2 - v_f^2 &= 2g\Delta h \\ v_f^2 &= v_i^2 - 2g\Delta h \\ v_f &= \sqrt{v_i^2 - 2g\Delta h} \end{aligned}$$

If $g = 9.81$ m/s ^{2} and Δh = 2 m, then

$$v_f = \sqrt{v_i^2 - 39.24}\,\mathrm{m/s} = \sqrt{v_i^2 - 508.6}\ \mathrm{km/h}$$

Plotting this relationship, we get:





Comments:

- Note that there was information given in the problem that we didn't really need, namely the mass of the train.
- The slower the train is going, the more significant the impact of the rise on its speed

---

Problem 10: Disc Brake Stopping Time and Distance

Solution 10.1: Jay's Solution

Given:

Truck: m = 1800 kg, v = 90 km/h

Truck with load: m = 1800+2250 kg, v = 60 km/h

Find: The time and distance that it takes for each to come to a stop, in terms of an identical stopping force $\boldsymbol{F}$ in each case. Compare.

Plan:

- Force applied over time is change in ,momentum, so we can use the change in momentum to find the stopping time
- Force applied over distance is change in kinetic energy, so we can use the change in energy to find the stopping distance.

Analysis:

$$F \cdot \Delta t = mv$$
$$\Delta t = \frac{mv}{F}$$

For truck: $mv = 1800 \cdot 90 = 162,000$

For truck with load: $mv = (1800 + 2250) \cdot 60 = 243,000$

Change in momentum of truck with load is greater, so stopping time is greater.

$$F \cdot \Delta x = \frac{1}{2} mv^2$$
$$\Delta x = \frac{mv^2}{2F}$$

For truck: $mv^2 = 1800 \cdot 90^2 = 14,580,000$

For truck with load: $mv^2 = (1800 + 2250) \cdot 60^2 = 14,580,000$

Change in kinetic energy is the same, so the stopping distance is the same.

Problem 11: Inflating a Football

Solution 11.1: Jay's Solution

Given:

$P_1 = 11$ PSI, $T_1 = 70^\circ$ F

$P_2 = 13$ PSI

Find:

T_2, assuming constant volume

Plan:

- Apply the ideal gas law $PV = nRT$ at both points
- Be sure to use temperature in K!

Analysis:

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$
$$T_2 = \frac{P_2}{P_1} T_1$$
$$= \frac{13}{11} 294.26$$
$$= 347.8^\circ\text{K}$$
$$= 166.3^\circ\text{F}$$

Comment:

If the pressure were truly raised instantaneously at constant volume, then the temperature in F would more than double! In practice, we couldn't raise the pressure instantaneously.

Problem 12: Pressure in a Balloon

Solution 12.1: Peter Nistler Solution

For all the sub-problems we can start with the equation: $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$.

(a) We are given: $P_1 = P_2$, $T_1 = 1.2 * T_2$. Solve for V_1.

$$V_1 = \frac{P_2 V_2 T_1}{P_1 T_2} = \frac{V_2 T_1}{T_2} = \frac{V_2(1.2 * T_2)}{T_2} = 1.2 * V_2$$

(b) We are given: $V_1 = V_2, P_1 = 1.15 * P_2$. Solve for T_1.

$$T_1 = \frac{P_1 V_1 T_2}{P_2 V_2} = \frac{P_1 T_2}{P_2} = \frac{(1.15 * P_2)T_2}{P_2} = 1.15 * T_2$$

(c) We are given: $T_1 = T_2, V_1 = 0.9 * V_2$. Solve for P_1.

$$P_1 = \frac{P_2 V_2 T_1}{T_2 V_1} = \frac{P_2 V_2}{V_1} = \frac{P_2 V_2}{0.9 * V_2} = \frac{P_2}{0.9} = 1.11 * P_2$$

Problem 13: Supporting a Weight with a Piston

Solution 13.1: Peter Nistler Solution

(a) $F = ma |\ m = 50kg, a = 9.81\frac{m}{s^2}$

$F = 490.5\ N$

(b) $p = \frac{F}{A} |\ F = 490.5N, A = \pi r^2 |\ r = 1m$

$p = 156.13\ Pa$

(c) $F = ma |\ m = 55kg, a = 9.81\frac{m}{s^2}$

$F = 539.55\ N$

$p = \frac{F}{A} |\ F = 539.55N, A = \pi$

$p = 171.74\ Pa$

Assume: $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$. We are given: $T_1 = T_2$.

We arrive at the simplified equation: $P_1V_1 = P_2V_2$

$V_2 = \frac{P_1V_1}{P_2} |\ P_1 = 156.13Pa, P_2 = 171.74Pa$

$V_2 = 0.909 * V_1$

We know that: $V_{cylinder} = h\pi r^2$.

$V_2 = h_2\pi r^2, V_1 = h_1\pi r^2$

$h_2\pi r^2 = 0.909 * h_1\pi r^2$

$h_2 = 0.909 * h_1$

Height changed by: $-0.091 * h_1$.

Problem 14: Units of Measure

Solution 14.1: Peter Nistler Solution

(a) Momentum = $\frac{kg*m}{s}$

(b) Momentum = $N * s$

(c) Force = $\frac{kg*m}{s^2}$

(d) Pressure = $\frac{kg}{m*s^2}$

(e) Pressure = $\frac{N}{m^2}$

(f) Momentum = $Pa * m^2 * s$

Problem 15: More Units of Measurement

Solution 15.1: Peter Nistler Solution

(a) $\text{Energy} = \frac{kg * m^2}{s^2}$

(b) $\text{Energy} = N * m$

(c) $\text{Work} = \frac{kg * m^2}{s^2}$

(d) $\text{Work} = N * m$

Problem 16: Stored Potential Energy

No solutions posted for this problem.

Problem 17: Hydroelectric Power Plant

Solution 17.1: Jay's Solution

Part 1

Given: dam height ($h = 50$ m), density of water ($\rho = 1000$ kg/m^3)

Find: potential energy (E) per unit volume

Plan:

- Determine the potential energy for some volume V of water at height h
- Divide by V to get the energy per unit volume

Analysis:

$$\begin{aligned} E &= mgh \\ &= (\rho V)gh \\ \frac{E}{V} &= \rho gh \\ &= (1000\frac{kg}{m^3})(9.81\frac{m}{s^2})(50m) \\ &= 490,500\frac{J}{m^3} \end{aligned}$$

Part 2

Given: dam height ($h = 50$ m),

Find: velocity of water (v) at outlet

Plan:

Use conservation of energy--a blob of water of mass m "lost" from the top of the dam must lead to a blob of the same mass exiting at the outlet at velocity v

Analysis:

$$\begin{aligned} mgh &= \frac{1}{2}mv^2 \\ v &= \sqrt{2gh} \\ &= \sqrt{2(9.81)(50)} \\ &= 31.32\ \frac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

Part 3

Given:

- Energy per unit volume from part 1
- Exit velocity from part 2
- Flow rate through the turbine of 2 cubic meters per second

Find: Power transmitted by the water to the turbine

Plan:

Power is energy per unit time, which we can determine from the Energy/Volume and the flow rate in Volume/Time

Analysis:

$$\begin{aligned} \text{Power} &= \frac{\text{Energy}}{\text{Volume}} \times \text{Flow Rate} \\ &= 490,500\ \frac{\mathrm{J}}{\mathrm{m}^3} \times 2\ \frac{\mathrm{m}^3}{\mathrm{s}} \\ &= 981\,\mathrm{kW} \end{aligned}$$

Problem 18: Ethanol Air-to-Fuel Ratio

Solution 18.1: Jay's Solution

Part 1

Given: The combustion equation for ethanol $C_2H_5OH + 3O_2 \rightarrow 2CO_2 + 3H_2O$

Find:
The stoichiometric air/fuel ratio, which is defined as the ratio of the mass of air to the mass of ethanol required for complete combustion

Plan:

The plan for solving this follows the same pattern as the plan for the air/fuel ration for gasoline from Example 4.8

1. Calculate the weight of 1 mol of ethanol
2. Calculate the weight of oxygen required to burn 1 mol of ethanol
3. Calculate the weight of air that contains the required amount
4. Divide the weight of the required air by the weight of 1 mol of ethanol

Atomic weights:

element:	H	C	O
atomic weight (g/mol):	1	12	16

Analysis:

1. Atomic weight of 1 mol of ethanol

mass of C_2H_5OH = 2(mass C) + 6(mass H) + 1(mass O)

$= 2(12) + 6(1) + 1(16)$

$= 46$ g

2. Mass of oxygen to burn 1 mol of ethanol

From the reaction, need 3 mols of O_2

Mass of O = $(3)(2)(16)$

$= 96$ g

3. Mass of air containing 96 g of oxygen

Assuming that air is 23.2% oxygen by mass

mass of air = $96/0.232$

$= 413.8$ g

4. Ratio of mass of air to mass of ethanol

$$\frac{413.8}{46} = 9$$

Thus the air to fuel ratio for ethanol is **9**

Part 2

Given:

- The energy densities for for both gasoline (45 MJ/kg) and ethanol (27 MJ/kg)
- Fuel efficiency of gasoline (12 km/l) for a certain automobile

Find: the fuel efficiency for the same car, assuming that it could be run on ethanol

Plan:

- Assume :
  - that the engine can burn ethanol as efficiently as it can burn gasoline.
  - that the car can travel the same distance for the same energy derived from burning either fuel
- Use ratio of energy densities to compare driving distances for equal masses of both fuels
- Look up the densities of both fuels and correct for the fact that equal masses of fuel may have different volumes
- Combine these results to get the fuel efficiency for ethanol

Analysis:

From the energy densities, a give mass of ethanol would propel a car 27/45 as far as an equal mass of gasoline

According to http://www.simetric.co.uk/si_liquids.htm, gasoline has a density of approximately 737 kg/m^3, while ethanol has a density of approximately 789 kg/m^3. Thus a given volume of ethanol would have a mass 789/737 times mass of an equal volume of gasoline

Combining these results, we get

$$\text{ethanol fuel efficiency} = 12\ \frac{\text{km}}{\text{l}} \times \frac{27}{45} \times \frac{789}{737} = 7.7\ \frac{\text{km}}{\text{l}}$$

Part 3:

Given:

- the stoichiometric air-fuel ratios of gasoline and ethanol (from first part of problem)
- the energy densities and densities of the two fuels (from the second part of the problem)

Find:

- the ratio of the energy produced by burning equal volumes of air-fuel mixtures of the two fuels at the stoichiometric air-fuel ratio

Plan:

1. Determine the mass of fuel in each mixture and then calculate their ratio
2. Use the energy density of the fuels to determine the energy produced

Analysis:

Let's assume we have a volume V of a mixture fuel F, at stoichiometric air-fuel ratio R_F, where the density of the fuel is ρ_F and the density of air is ρ_A. We can calculate the mass m_F of fuel in the mixture as follows:

$$V = V_A + V_F = \frac{m_A}{\rho_A} + \frac{m_F}{\rho_F}$$

Since $R_F = m_A/m_F$, $m_A = m_F R_F$, and so

$$V = \frac{m_F R_F}{\rho_A} + \frac{m_F}{\rho_F}$$

Solving for m_F, we get

$$m_F = \frac{\rho_F \rho_A V}{\rho_F R_F + \rho_A} = \frac{\rho_A V}{R_F + \frac{\rho_A}{\rho_F}}$$

In the denominator of this expression, R_F is greater than 1 and ρ_A/ρ_F is much less than 1, so we can simply as

$$m_F = \frac{\rho_A V}{R_F}$$

Thus the ratio of the masses of ethanol m_E and gasoline m_G in equal volumes V of mixtures at their air-fuel ratios R_E and R_G is

$$\frac{m_E}{m_G} = \frac{R_G}{R_E}$$

Now, we can calculate the ratio of the energies E_E and E_G, where the fuels have energy densities D_E and D_G as

$$\begin{aligned}\frac{E_E}{E_G} &= \frac{m_E}{m_G}\frac{D_E}{D_G}\\ &= \frac{R_G}{R_E}\frac{D_E}{D_G}\\ &= \frac{14}{9}\frac{27}{45}\\ &= 0.93\end{aligned}$$

Comments:

From this calculation the energy produced by the ethanol is 0.93 times the energy produced by gasoline, which is a little bit less. We note that ethanol is often preferred over gasoline as a racing fuel. The reasons for this go beyond just the energy values. Ethanol has a higher octane rating than gasoline, which means it can stand a higher compression ratio. It also has a higher latent heat, which means that it does a better job of cooling the engine when a mix is injected before ignition. See http://www.drivingethanol.org/motorsports/racing_fuel_characteristics.aspx for a good explanation.

Problem 19: Slow Versus Fast Combustion in a Piston Engine

No solutions posted for this problem.

Problem 20: Pump Forces

Solution 20.1: Jay's Solution

The forces would be the same in this case. This is because the force is equal to the water pressure on the piston times the area of the piston. The pressure on the piston depends on the depth of the water above the piston, but does not depend on the *shape*. Since the depth is the same in each case, the pressure is the same.

Problem 21: Water Pressure

Solution 21.1: Jay's Solution

Part 1

Given: Depth $h = 10$ m of a column of water

Find: The pressure P at the base of the column

Plan:
The problem should be clearer as to whether we want the pressure just inside the column of water or outside the column. The pressure outside is atmospheric pressure--inside the column is what we really want. We can get this using the methods from Section 4.4.1.

Analysis:

$$\begin{aligned} P &= \rho g h \\ &= (1000\,\frac{\mathrm{kg}}{\mathrm{m}^3})(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2})(10\,\mathrm{m}) \\ &= 98.1\,\mathrm{kPa} \end{aligned}$$

Part 2

Given: Height $h = 10$ m of a column of water

Find: The velocity that water will flow out of the bottom of the column

Plan: Use the principle of conservation of energy

- The change in potential energy of a blob of mass m being removed from the top of the column must be the same as the kinetic energy of a blob of the same mass flowing out of the column at velocity v

Analysis:

$$\begin{aligned} mgh &= \frac{1}{2} m v^2 \\ v &= \sqrt{2gh} \\ &= \sqrt{2(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2})(10\,\mathrm{m})} \\ &= 14.0\,\frac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

Problem 22: Hydraulic Machines

Solution 22.1: Jay's Solution

Part 1

If the fluid is incompressible, then according to conservation of mass, the volume of oil being squeezed out of the 2 cm cylinder must be the same as the volume entering the 5 cm cylinder. The volume V of a cylinder of diameter d and length x is:

$$V = \pi(\frac{d}{2})^2 x$$

Thus

$$\begin{aligned} \pi(\frac{d_{\mathrm{small}}}{2})^2 x_{\mathrm{small}} &= \pi(\frac{d_{\mathrm{large}}}{2})^2 x_{\mathrm{large}} \\ x_{\mathrm{large}} &= x_{\mathrm{small}} \frac{d^2_{\mathrm{small}}}{d^2_{\mathrm{large}}} \\ &= (1)(\frac{4}{25}) \\ &= 0.16\,\mathrm{cm} \end{aligned}$$

Part 2

According to conservation of energy, the work done at either end of the system must be the same. Since work is force times distance:

$$(50\,\mathrm{N})(1\,\mathrm{cm}) = F(\frac{4}{25}\,\mathrm{cm})$$
$$F = 50(\frac{25}{4})$$
$$= 312.5\,\mathrm{N}$$

Part 3

The pressure distribution must be uniform throughout the oil (neglecting the minute pressure difference due to the change in depth).

Part 4

The key observation from this example is that a small force applied over a distance of 1 cm at one end of the system transformed into a larger force over a smaller distance at the other end of the system. This *hydraulic machine* thus provides the same kind of functionality that machines such as a lever or block-and-tackle do. An advantage of a hydraulic machine is that the tube connecting the two cylinders doesn't have to be in a straight line, which makes it more flexible than the other approaches. This is precisely the type of machine used in many automobile brake systems. Stepping on the brake pedal with a relatively small force moves the piston in the pedal cylinder, forcing brake fluid through the brake line into the brake cylinder, which has a larger diameter. This multiplies the force that the piston in the brake cylinder can apply to stopping the car. See the article in "How Stuff Works" on "How Brakes Work" (http://auto.howstuffworks.com/brake.htm) for a good explanation.

Part 5

There are many other systems that use hydraulic machines. Examples include construction or earth-moving equipment, such as front-end loaders or dump trucks.

Chapter 5

Chapter 5

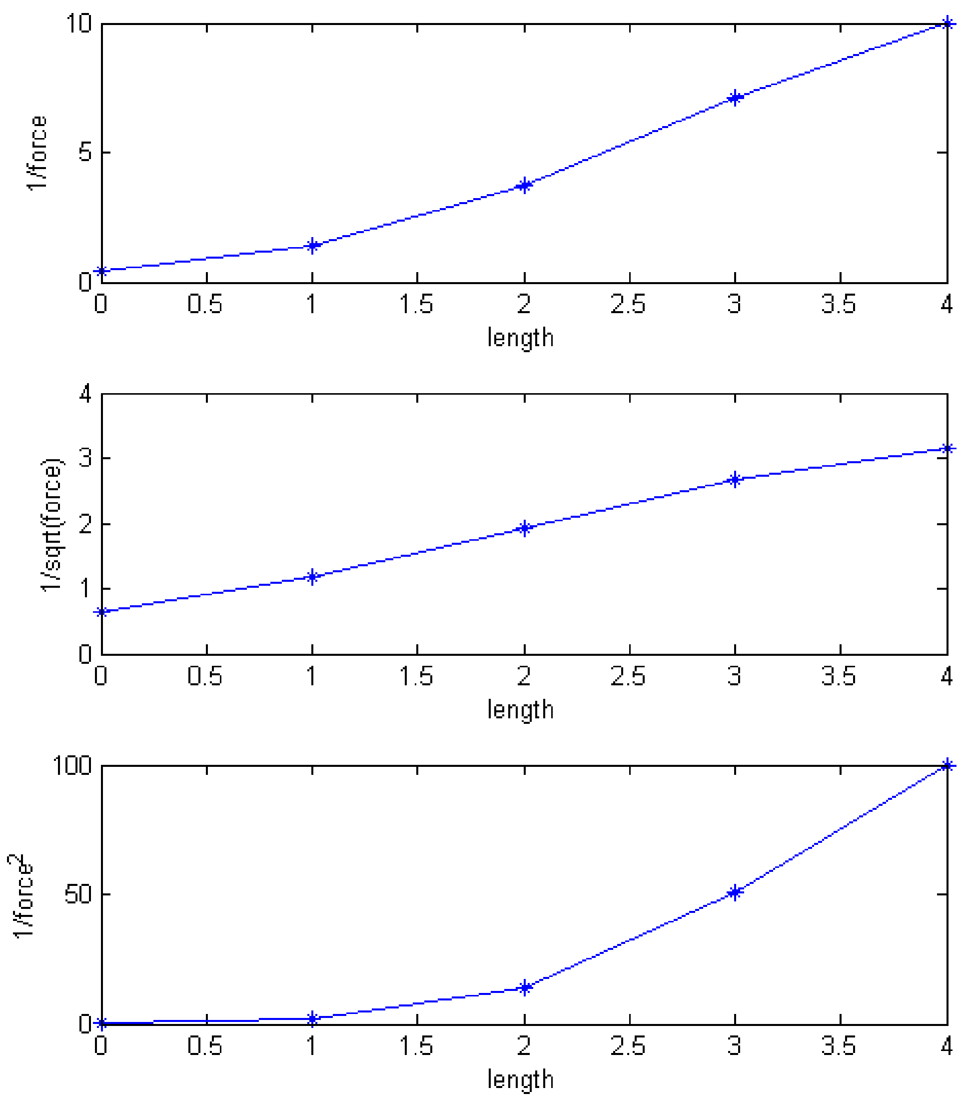
Problem 1: Fitting Theory to Data for an Unknown Spring

Solution 1.1: Jay's Solution

Part 1

Below are plots of

- $1/\text{force}$ vs. length
- $1/\sqrt{\text{force}}$ vs. length
- $1/\text{force}^2$ vs. length



Visually, it appears that the best fit is for $1/\sqrt{\text{force}}$ vs. length, so we can say that the best theory is that length is inversely proportional to the square root of force.

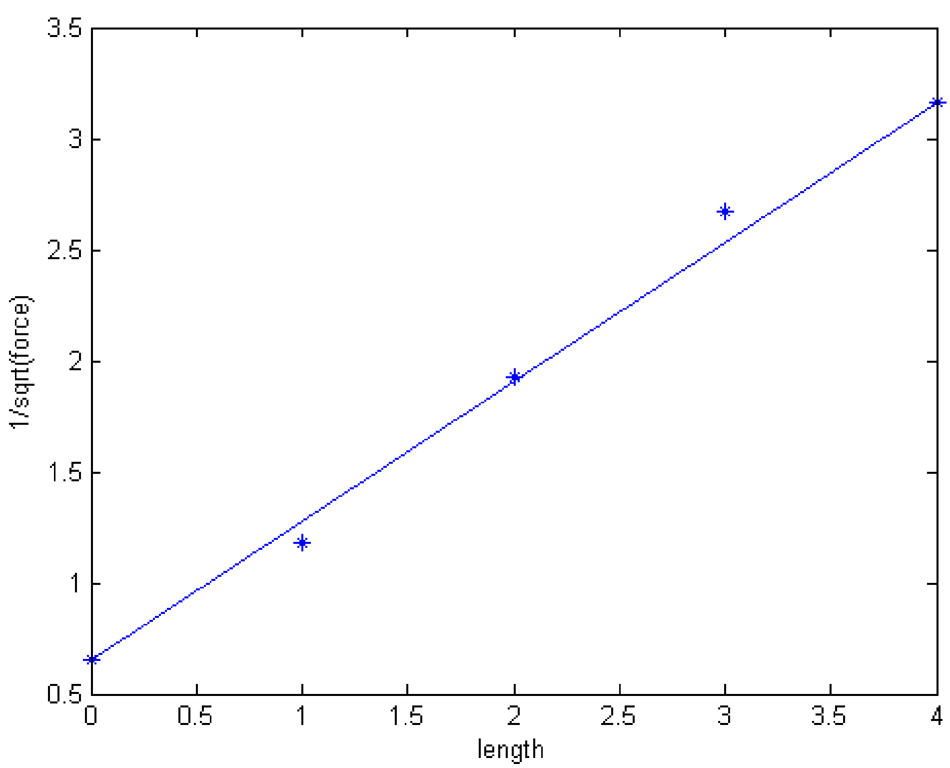
This plot was produced with the MATLAB script below

```
length = 0:4;
force = [2.36    0.72    0.27    0.14    0.10];
subplot(3,1,1)
plot(length,1./force,'-*')
xlabel('length');
ylabel('1/force');
subplot(3,1,2)
plot(length,1./sqrt(force),'-*')
xlabel('length');
ylabel('1/sqrt(force)')
subplot(3,1,3)
plot(length,1./force.^2,'-*')
xlabel('length');
ylabel('1/force^2')
```

Part 2

Given the plot of $1/\sqrt{\text{force}}$ vs. length, we'll now estimate the best fit line and find its slope and intercept. We'll estimate the best fit line as passing through the first and last points in the data set, or

length	force	1/sqrt(force)
0	2.36	0.65
4	0.10	3.16



The slope is

$$m = \frac{3.16 - 0.65}{4 - 0} = .63$$

The intercept is 0.65 (corresponding to where length = 0)

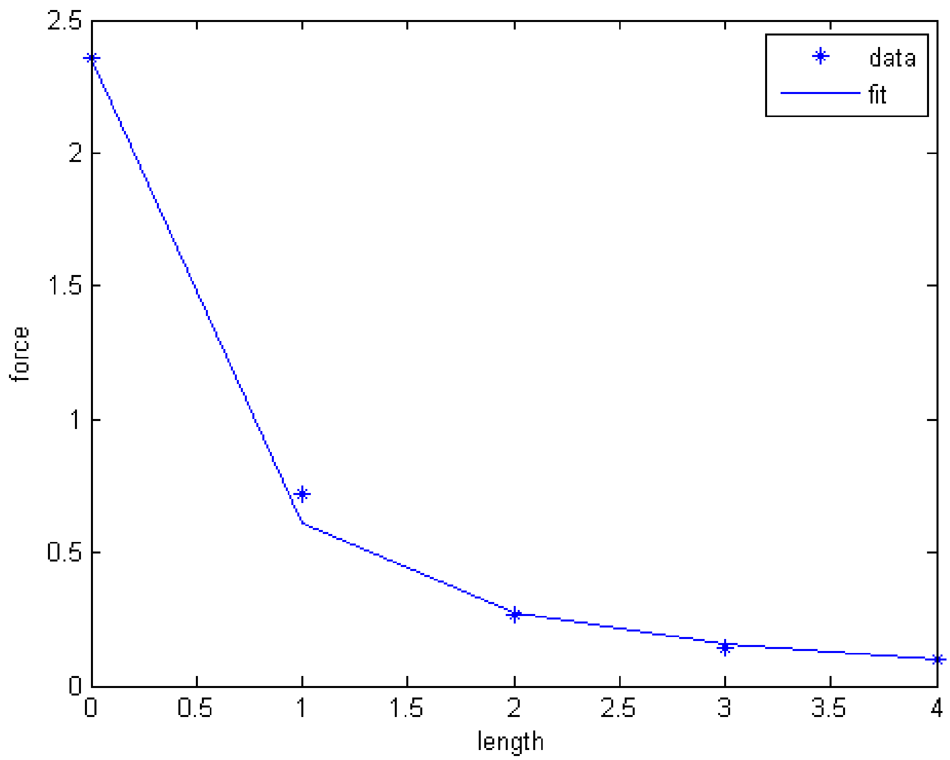
Thus the equation of the best fit line for force F and length L is

$$\begin{aligned}\frac{1}{\sqrt{F}} &= mL + b \\ &= 0.63L + 0.65\end{aligned}$$

Rewriting this as force as a function of length, we get

$$\begin{aligned}F &= (\frac{1}{mL + b})^2 \\ &= (\frac{1}{0.63L + 0.65})^2\end{aligned}$$

A plot of this equation, along with the original data, is as follows:



This plot was generated using the following MATLAB code:

```
z = 1./sqrt(force)
m = (z(5)-z(1))/(length(5)-length(1))
b = z(1) - m*length(1)
force_pred = (1./(m*length + b)).^2;
figure
plot(length,force,'*')
xlabel('length')
ylabel('force')
hold on
plot(length,force_pred)
legend('data','fit')
hold off
```